STRATEGIC BUYERS AND
PRIVATELY OBSERVED PRICES

BY

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NOTES, COMMENTS, AND LETTERS TO THE EDITOR

Strategic Buyers and Privately Observed Prices[1]

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A model of finitely repeated price competition between two sellers with differentiated goods and a large buyer is analyzed. The set of pure strategy sequential equilibria is investigated under public and private monitoring. With private monitoring, i.e., when prices are not observable to the competing sellers, all sales are made by the better seller and the set of repeated game equilibrium payoffs coincides with the stage game subgame perfect equilibrium payoffs. This is in sharp contrast to the game with perfect monitoring where the folk theorem obtains. *Journal of Economic Literature* Classification Numbers: C72, D43.   © 2002 Elsevier Science (USA)

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1. INTRODUCTION

This note considers the informational role of past prices in a model of repeated price competition for large buyers. The basic model is a finite repeated extensive form game between two differentiated sellers and a single buyer with a unit demand in each period. At the beginning of each stage the sellers set prices, and the buyer selects the seller after having observed the prices. The competing sellers may or may not observe the prices offered by their competitors.

   We analyze the effects of informational constraints imposed on the repeated game. The case of perfect monitoring is compared with private monitoring where the sellers observe no signal of their opponent's price.[2] As a consequence, with private monitoring the identity of a deviator may not be common knowledge among the players. Our main result is that in contrast to the games with perfect monitoring where the folk theorem applies, the sequential equilibrium payoff set shrinks to the stage game equilibrium payoff set in finitely repeated games with private monitoring. First of all, sales are made only by the seller with the better product. Second, bounds are obtained for the equilibrium payoffs to the buyer and the sellers in terms of the outside option that the buyer has in the form of the less efficient seller. The basic intuition for the result is quite simple and we show that it can be extended beyond the case of unit demand. In models of price competition, the firms punish deviations by low future prices. Since the buyer in our model is forward looking, these punishments are beneficial to her and as the identity of the deviator is not common knowledge, a deviation by the buyer would also result in low future prices. Our result shows that when the incentives of the sellers as well as the buyer are maintained simultaneously, the set of equilibrium payoffs shrinks considerably.

   This reduction in the equilibrium payoff set is caused by the conjunction of three elements in our model. The first essential ingredient to the model is fully transferable utility between the sellers and the buyer. We show by an example that if the prices offered by the sellers are restricted to be positive, then the game with private monitoring may have a larger payoff set than the game with perfect monitoring. The second requirement is that there be a single large buyer in the market. We show that with multiple buyers, the set of equilibrium payoffs increases, and in the limit as the number of buyers tends to infinity, the entire feasible set can be approximated by equilibrium payoffs. The third requirement is that the game have a finite time horizon. For an infinite horizon game, we construct examples of equilibria that violate our finite horizon payoff bounds in Bergemann and Välimäki [4]. Since the multiplicity of equilibria in the game with perfect monitoring is created through the use of weakly dominated strategies, we also give an extension of the model to the case where the buyer has a downward sloping demand. In this case, the stage game has many equilibria that survive standard refinements such as trembling hand perfection and yet all the repeated game equilibrium payoffs are convex combinations of stage game equilibrium payoffs. In other words, the folk theorem fails for these games as well.

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[2] We use the term private monitoring rather than imperfect monitoring, in order to emphasize that there is no randomness in the monitoring technology as in most models of imperfect monitoring.

A number of papers have analyzed the problem of repeated competition under imperfect monitoring. Starting with Green and Porter [9], a popular approach has been one in which the actions taken by the sellers generate a publicly observable signal. All players may then condition their continuation play on this signal. This line of work culminates in the papers by Abreu *et al.* [1] and Fudenberg *et al.* [8], where it is shown that as long as a minimal statistical requirement on the quality of the signal is satisfied, a collusive arrangement may be supported in a perfect public equilibrium. A number of recent papers including Sekiguchi [11] and Bhaskar and van Damme [5] question the appropriateness of assuming the existence of public signals. If a common signal can be constructed through, e.g., preplay communication, then Kandori and Matsushima [10] and Compte [6] show that collusive equilibria are possible in games with imperfect private monitoring while Ben-Porath and Kahneman [2] obtain the result for noiseless private monitoring. A recent contribution by Compte [7] considers price competition for a large buyer with secret price cutting in a two-stage model with communication. To our knowledge, the current note is the first attempt to analyze a repeated game with an extensive form stage game in the context of private monitoring.

2. PERFECT MONITORING

Two sellers, $j \in \{1, 2\}$, sell a product to a single buyer with unit demand repeatedly over a finite time horizon. The maximum amount that the buyer is willing to pay for seller j's product is denoted by v_j for $j \in \{1, 2\}$. At the beginning of period $t \in \{0, 1, ..., T\}$, the sellers announce simultaneously prices p_j^t and the buyer chooses between the sellers. The buyer's net payoff in period t is given by $u_j^t \triangleq v_j - p_j^t$ when buying from seller j. We normalize the production cost to zero so that the per period payoff to the seller is equal to the revenue. All players maximize their expected sum of payoffs.

Consider first the benchmark case of full information. When deciding her price in period t, each seller $j \in \{1, 2\}$ has observed history $h_j^t = \{p_1^0, p_2^0, d^0, ..., p_1^{t-1}, p_2^{t-1}, d^{t-1}\}$ where $d^s \in \{1, 2, R\}$ is the buyer's choice of the seller in period s, and R denotes the rejection of both sellers' offers. In addition to the history available to the two sellers in period t, the buyer also knows the period t prices chosen by the two sellers and thus her information is given by $h_b^t = h_j^t \cup \{p_1^t, p_2^t\}$. The (behavior) strategies in period t available to seller j are then functions from the set of possible period t histories, H_j^t, to real numbers. The buyer's (behavior) strategies in period t are functions from all possible period t histories H_b^t to $\{1, 2, R\}$.

Denote the T-fold repetition of the stage game by $\Gamma(T)$. Observe first that the stage game has a continuum of subgame perfect equilibrium payoffs

if $v_1 \neq v_2$. Without loss of generality, let $v_1 \geqslant v_2$ and define $\Delta v = v_1 - v_2$.[3] Any pair of prices, (p_1, p_2) such that $\Delta v \geqslant p_1 \geqslant 0$, $p_2 = p_1 - \Delta v$, and buyer's strategy,

$$d = \begin{cases} 1 & \text{whenever} \quad p_2 \geqslant p_1 - \Delta v \quad \text{and} \quad v_1 \geqslant p_1, \\ 2 & \text{whenever} \quad p_2 < p_1 - \Delta v \quad \text{and} \quad v_2 \geqslant p_2, \\ R & \text{otherwise}, \end{cases} \quad (1)$$

is a subgame perfect equilibrium of this extensive form game.[4] Denote the buyer's stage game strategy described above by d^*. Notice that in all stage game equilibria, seller 1 makes the sales, and as a consequence, seller 2 has a surplus of 0. Since we have multiple stage game payoff vectors, the continuation payoffs can be made dependent on the actions chosen, and there is a chance that a wide variety of outcomes might be supportable in subgame perfect equilibrium.

The closure of the set of individually rational feasible (average) payoff vectors in the stage game is given by

$$U^F = \left\{ (u_1, u_2, u_b) \in \mathbb{R}^3_+ \mid u_1 + u_2 + u_b \leqslant v_1 \right\}.$$

Apart from dealing with an extensive form stage game, the stage game subgame perfect equilibria are not distinct in payoffs for seller 2. Hence we cannot appeal directly to the result in Benoit and Krishna [3] to conclude the existence of collusive equilibria in this game. Our first result shows that the set of equilibrium payoffs in $\Gamma(T)$ converges to U^F as $T \to \infty$. Strategies reminiscent to those in Benoit and Krishna [3] are used to establish this.

PROPOSITION 1. *Fix $u \in U^F$ and $\epsilon > 0$. Then there is a $\hat{T}$ such that whenever $T \geqslant \hat{T}$, $\Gamma(T)$ has a subgame perfect equilibrium (in pure strategies) with the average payoffs of the players in $B_\epsilon(u)$.*

Proof. See Bergemann and Välimäki [4]. ∎

3. PRIVATE MONITORING

In this section, we consider the game with private information about past moves. In particular, we assume that each p_j^t is observable to seller j and

[3] If $v_1 = v_2$, then the stage game has a single subgame perfect equilibrium payoff, and by backward induction, the finitely repeated game has a unique payoff vector as well.

[4] Observe that the equilibria with $p_2 < 0$ would fail refinements such as elimination of weakly dominated strategies or trembling hand perfection (when considering a suitably discretized version of the model). In the next section, we extend our model to cover situations where there are multiple trembling hand perfect equilibria.

the buyer, but not to the other seller. The buyer's information sets are unchanged from the full information case, but the sellers have less information available: $h_j^t = \{p_j^0, d^0, ..., p_j^{t-1}, d^{t-1}\}$ for $j \in \{1, 2\}$. Let $H_p^t = H_1^t \cap H_2^t$ denote the public history in period t, where we notice that the part of history that is common knowledge between the players is the sequence of purchases made by the buyer. The (behavior) strategies are then sequences of functions, $\mathbf{p}_j = \{p_j^t\}_{t=0}^T$, where $p_j^t: H_j^t \to \mathbb{R}$, and $\mathbf{d} = \{d^t\}_{t=0}^T$, where $d^t: H_b^t \to \{1, 2, R\}$. In what follows, we look for pure strategy sequential equilibria of this game.

Let $(\mathbf{p}_1, \mathbf{p}_2, \mathbf{d})$ be a sequential equilibrium of the repeated game with private monitoring. Recall that the buyer monitors the prices given by the sellers perfectly. We first want to argue that it is sufficient to consider deviations by a single seller to reach the conclusion that the set of repeated game sequential payoffs must coincide with the set of stage game equilibrium payoffs. The following lemma is the key to our main result. It shows that along the equilibrium path, the coalition consisting of seller j and the buyer can always guarantee a payoff of at least v_j per period.

LEMMA 1. *In any sequential equilibrium $(\mathbf{p}_1, \mathbf{p}_2, \mathbf{d})$ of the game with private monitoring, the sum of the average equilibrium payoffs to seller j and the buyer is at least v_j.*

Proof. Denote the opponent of seller j by k and denote the equilibrium continuation payoffs to the players at their information sets by $V_j^t(h_j^t)$, $V_k^t(h_k^t)$, and $V_b^t(h_b^t)$.[5] In addition, let $V_b^t(h_b^t \mid \mathbf{p}_k)$ be the value function of the buyer resulting from the dynamic programming problem where the buyer is restricted to either choose k or R in each period. The value function can be obtained by backward induction:

$$V_b^T(h_b^T \mid \mathbf{p}_k) = \max_{d^T \in \{k, R\}} \{u_{d^T}^T\},$$

where we recall that $u_k^t = v_k - p_k^t$ and $u_R^t = 0$. The general recursion is:

$$V_b^t(h_b^t \mid \mathbf{p}_k) = \max_{d^t \in \{k, R\}} [u_{d^t}^t + V_b^{t+1}(h_b^t, d^t \mid \mathbf{p}_k)], \text{ for } t \in \{0, ..., T-1\}.$$

Observe that when seller k plays according to her equilibrium strategy $\mathbf{p}_k$, the buyer can always guarantee himself a continuation payoff $V_b^t(h_b^t \mid \mathbf{p}_k)$ after seller k has observed history h_k^t (and recall that the buyer knows h_k^t). Let $H^t(j)$ denote the set of histories where at most player j has deviated in

any period prior to t. As long as the sequence $\{d^t\}_{t=0}^T$ is consistent with the other players' equilibrium strategies, player j believes that k is following $\mathbf{p}_k$. In fact, this belief is common knowledge between the buyer and seller j in any sequential equilibrium.

Consider then a history in $H^T(j)$. Since j knows $p_k^T(h_k^T)$, a simple and standard undercutting argument shows that $V_j^T(h_j^T) + V_b^T(h_b^T) \geqslant v_j$ for all h_j^T, h_b^T consistent with some $h^T \in H^T(j)$. The same undercutting argument shows that in fact $V_b^T(h_b^T) = V_b^T(h_b^T \mid \mathbf{p}_k)$. Assume next that $V_j^t(h_j^t) + V_b^t(h_b^t) \geqslant (T-t)\, v_j$ and $V_b^t(h_b^t) = V_b^t(h_b^t \mid \mathbf{p}_k)$ for all $h^t \in H^t(j)$ and consider an $h^{t-1} \in H^{t-1}(j)$. To see that $V_j^{t-1}(h_j^{t-1}) + V_b^{t-1}(h_b^{t-1}) \geqslant (T-t+1)\, v_j$, observe that if $d^t = j$, then the inequality is satisfied by the induction hypothesis, and if $d^t \neq j$, then $V_j^{t-1}(h_j^{t-1}, d^t) + V_b^{t-1}(h_b^{t-1}, d^t) \geqslant V_j^{t-1}(h_j^{t-1}, j) + V_b^{t-1}(h_b^{t-1}, j) \geqslant (T-t+1)\, v_j$ where the first inequality follows from the full transferability of utility in period $t-1$ between seller j and the buyer. Finally, the equality $V_b^{t-1}(h_b^{t-1}) = V_b^{t-1}(h_b^{t-1} \mid \mathbf{p}_k)$ follows again from an undercutting argument. ∎

With the lemma in place, we can prove the main result of this note.

THEOREM 1. *In the repeated game with private monitoring, the set of pure strategy sequential equilibrium payoffs coincides with the set of stage game subgame perfect equilibrium payoffs.*

Proof. Let (u_1, u_2, u_b) denote an arbitrary sequential equilibrium average payoff. By individual rationality, $u_1 \geqslant 0$, $u_2 \geqslant 0$ and $u_b \geqslant 0$. Since any history h^t on the equilibrium path belongs to $H^t(1) \cap H^t(2)$, we can use the lemma above to show that $u_1 + u_b \geqslant v_1$ and $u_2 + u_b \geqslant v_2$. Finally, by feasibility $u_1 + u_2 + u_b \leqslant v_1$. Combining these, we get $(u_1, u_2, u_b) = (u_1, 0, v_1 - u_1)$ for some $u_1 \in [0, \Delta v]$. ∎

The economic significance of this result is immediate. If the buyer prefers the product of seller 1 to the product of seller 2, then the equilibrium price of the efficient seller is between 0 and the quality difference in all periods, and the efficient seller makes all the sales in the model. As a result, we see that the possibilities for collusion are severely limited by the unobservability of opponents' prices in the finite horizon model.

We would like to stress two aspects of the above result. First of all, if prices are privately observed between each seller and the buyer, it is impossible for the outsider to identify which of the parties deviated. This fact coupled with the transferability of utility makes it possible for each seller and the buyer to act as a colluding coalition. As a result, the game can be analyzed as one where the effective players are the coalitions formed by an individual seller and the buyer. This leads to the reasoning behind Lemma 1. The second observation is that the set of equilibrium payoffs is the same as the set of perfect public equilibrium payoff vectors. As shown in Bergemann and Välimäki [4], this observation does not hold in the

infinite horizon game. The reason for this is that the first step in the induction argument leading to the proof of Lemma 1 is no longer available.

In order to demonstrate that the assumption of full transferability of utility between each seller and the buyer is crucial to our conclusion, consider the equilibria in a two period version of the model, where the prices of the two firms are restricted to be nonnegative in each period. Observe first that in this case, the stage game equilibrium payoff is unique, i.e., $(\Delta v, 0, v_2)$ and as a result, the subgame perfect equilibrium payoff in the game with perfect monitoring is also unique. With private monitoring, the set of equilibrium payoffs is, however, strictly larger. The following strategies form part of a sequential equilibrium in the two period game, in which seller 1 makes all the sales in equilibrium

$$p_1^0 = v_1 - \frac{(v_2)^2}{v_1},$$

$$p_2^0 = 0,$$

$$d^0 = \begin{cases} 1, & \text{if } p_1^0 - p_2^0 \leqslant v_1 - \dfrac{(v_2)^2}{v_1} & \text{and} & p_1^0 \leqslant v_1, \\[3mm] 2, & \text{if } p_1^0 - p_2^0 > v_1 - \dfrac{(v_2)^2}{v_1} & \text{and} & p_2^0 \leqslant v_2, \\[3mm] R, & \text{otherwise,} \end{cases}$$

and in period $t = 1$

$$p_1^1 = \begin{cases} v_1 - v_2, & \text{if } \begin{cases} p_1^0 = v_1 - \dfrac{(v_2)^2}{v_1} & \text{and} & d^0 = 1, \text{ or} \\[3mm] p_1^0 \neq v_1 - \dfrac{(v_2)^2}{v_1}. \end{cases} \\[8mm] v_1, & \text{if } p_1^0 = v_1 - \dfrac{(v_2)^2}{v_1} & \text{and} & d^0 \in \{2, R\}. \end{cases}$$

$$p_2^1 = \begin{cases} 0, & \text{if } p_2^0 = 0, \\ v_2, & \text{if } p_2^0 \neq 0. \end{cases}$$

$$d^1 = \begin{cases} 1, & \text{if } p_1^1 - p_2^1 \leqslant v_1 - v_2 & \text{and} & p_1^1 \leqslant v_1, \\ 2, & \text{if } p_1^1 - p_2^1 > v_1 - v_2 & \text{and} & p_2^1 \leqslant v_2, \\ R, & \text{otherwise.} \end{cases}$$

In order to complete the description of the sequential equilibrium, we need to specify the consistent beliefs that make the strategies above sequentially rational. The key information set is the one where seller 2 has

observed $d^0 = 2$. (Observe that the strategy of seller 2 in $t = 1$ is not contingent on the decision of the buyer.) Consider trembles by seller 1 and the buyer such that

$$\Pr \left\{ p_1^0 > v_1 - \frac{(v_2)^2}{v_1} \right\} = \varepsilon$$

and

$$\Pr \left\{ d^0 = \left| p_1^0 = v_1 - \frac{(v_2)^2}{v_1} \right. \right\} = \varepsilon^2.$$

Then as $\varepsilon \to 0$, seller 2 believes with probability 1 that seller 1 sets $p_1^1 = \Delta v$. Hence $p_2^1 = 0$ is a best response at the information set following $d^0 = 2$ and $p_2^0 = 0$. The beliefs of seller 1 are as follows. If

$$p_1^0 = v_1 - \frac{(v_2)^2}{v_1}$$

and $d^0 = 2$, seller 1 believes that $p_2^0 = 0$ with probability v_2/v_1 and as a consequence, seller 1 expects seller 2 to set $p_2^1 = 0$ with probability v_2/v_1 and $p_2^1 = v_2$ with the complementary probability. As a result, $p_1^1 = v_1$ is a best reply. The buyer's strategy is clearly sequentially rational, and as the buyer is indifferent in $t = 0$, seller 1 must be extracting maximal surplus from the buyer given the strategy of seller 2. Since prices are restricted to be nonnegative, seller 2 has no profitable deviations either. Observe, however, that if the prices were not restricted, seller 2 would deviate to a negative price.

The equilibrium payoff in this game is

$$\left(2v_1 - v_2 \left(1 + \frac{v_2}{v_1} \right), 0, v_2 \left(1 + \frac{v_2}{v_1} \right) \right).$$

The better of the two sellers is obtaining a higher payoff (per period) than in the stage game equilibrium. It would also be easy to generate similar equilibria where seller 2 has a strictly positive expected payoff on the equilibrium path.

In this section, we make use of weakly dominated strategies in order to create multiple equilibria for the repeated game with perfect monitoring. Such equilibria would not survive standard refinements of Nash equilibrium such as an appropriate extension of trembling hand perfection. In order to show that our results do not depend critically on this feature of the model, we consider the case of multi-unit demand in the next section.

In that situation, the stage game has a multiplicity of undominated equilibria and yet the folk theorem fails.

4. EXTENSIONS

In this section, we outline two extensions of the model. The first considers the case of a buyer with a demand for more than a single unit in each period, and the second extension covers the case of multiple buyers.

4.1. *Multi-unit Demand*

Suppose that the buyer's demand function for the two products is as follows. Seller 1's product is worth v_1 to the buyer, seller 2's product is worth v_2 and the two products together are worth v_{12}. Assume that $v_1 + v_2 > v_{12} > v_1 > v_2$. The sellers compete in prices as follows. They submit two price quotes in each period, seller j charges p_j^t for seller j's product alone, and q_j^t if the buyer purchases both products. As before, assume that the prices are observable to the buyer only, but not to the competing seller, but that the purchase decisions, with $d^t \in \{1, 2, 12, R\}$, are observable for both sellers.[6]

It is easy to verify that the stage game has two connected components of stage game equilibrium payoffs. In the first, denoted by E_{12}, the buyer purchases both products and in the second, denoted by E_1, the buyer purchases seller 1's product only. The payoffs in these two components are given by

$$E_1 = \{(p_1, 0, v_1 - p_1)\},$$

where $p_1 \in [0, \Delta v]$, and

$$E_{12} = \{(q_1, q_2, v_{12} - q_1 - q_2)\},$$

where $q_1 \in [0, v_{12} - v_2]$, and $q_2 \in [0, v_{12} - v_1]$. Observe that most of these equilibria are undominated.

The convex hull of stage game equilibrium payoffs is then given by

$$U^S = \mathrm{co}(E_1 \cup E_{12}).$$

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[6] An alternative economic interpretation for this game is that two potential distributors are bidding for the right to sell a manufacturer's product. The possible outcomes in the game would then be an exclusive deal worth v_j to distributor j or a joint contract to the two distributors creating value $\frac{1}{2} v_{12}$ to each of them. The stage payoff vectors in this interpretation would be $(v_1 - p_1, 0, p_1)$, $(0, v_2 - p_2, p_2)$ and $(\frac{1}{2} v_{12} - q_1, \frac{1}{2} v_{12} - q_2, q_1 + q_2)$ for the three possible allocations where p_j is the bid price for an exclusive contract and q_j is the bid price of a joint contract.

We claim that the set of average equilibrium payoffs in the finitely repeated game with private monitoring coincides with U^S. If the game is repeated sufficiently many times, then any (per period) payoff in U^S can be approximated by a suitably chosen sequence of history independent stage game equilibria. To see that no other payoffs are consistent with sequential equilibrium, observe first that according to the same logic as in the previous section, the coalition of seller i and the buyer can guarantee a joint payoff of v_i in the last period of the game if at most seller i has deviated along the path of play. Based on this observation, the inductive argument used in Lemma 1 of the previous section can be used to conclude that seller i and the buyer can guarantee a joint payoff of v_i in any continuation following a history where at most seller i has deviated. This argument shows that in any sequential equilibrium, $u_i + u_B \geqslant v_i$. Feasibility requires that $v_{12} \geqslant u_1 + u_2 + u_B$. The claim is then established by observing that

$$U^S = \{(u_1, u_2, u_B) \in \mathbf{R}_+^3 \mid u_1 + u_2 + u_B \leqslant v_{12}, \, u_1 + u_B \geqslant v_1, \, u_2 + u_B \geqslant v_2\}.$$

The set of sequential equilibrium payoffs is thus limited again as a result of the imperfect observability of the actions by the sellers. As in the previous sections, transferability of payoffs together with private observability of the prices makes the coalitions consisting of a single seller and the buyer rather than the individual players the relevant units of analysis. The result in this section is somewhat different in its nature from the single unit case. There, we were able to conclude that the path of play in a repeated game consists of a sequence of stage game Nash equilibria. Here, we cannot reach such conclusion. This is demonstrated in the following two period example. Let $v_1 = 5$, $v_2 = 4$, $v_{12} = 6$, and consider second period continuation payoff vectors $(0, 0, 6)$ and $(1, 0, 4)$. Observe that both of these vectors arise in a stage game Nash equilibrium. Suppose that the equilibrium resulting in the first vector is to be played if $d^0 = \{12\}$, and the equilibrium resulting in the second is to be played if $d^0 \neq \{12\}$. The following strategies form an equilibrium in the first period: $p_1^0 = 5$, $p_2^0 = 4$, $q_1^0 = 4$, $q_2^0 = 2$, $d^0 = \{12\}$. Notice that the joint payoff to seller 1 and the buyer is 10 and the joint payoff to seller 2 and the buyer is 8 and as a result, the per period payoff vector is in U^S. Nevertheless, the play in the first period is not consistent with a stage game Nash equilibrium.

4.2. *Multiple Buyers*

We conclude this section with an extension to $I \geqslant 2$ strategic buyers. The products are assumed to have the same value for all buyers. We assume that the sellers quote separate prices for the buyers and that the prices are not observable to outsiders, i.e., other buyers and other sellers. We maintain the assumption that all purchasing decisions are publicly observable.

Denote seller j's price quoted to buyer i in period t by p_{ij}^t for $j \in \{1, 2\}$ and $i \in \{1, ..., I\}$. Denote the purchasing decision of buyer i in period t by d_i^t. The key difference to the single buyer case is that the buyers do not internalize their impact on the continuation values of the other buyers when contemplating a deviation. As a result, the earlier argument is weakened as not all future losses to a seller from breaking the collusive agreement are recorded as gains to the individual buyer. We demonstrate the effect of adding more buyers to the model by showing how the inefficient seller can achieve an increasing fraction of the total surplus as the number of buyers increases.

PROPOSITION 2. *For any $u \in U^F$ and for any $\varepsilon > 0$, there are $\hat{I} < \infty$ and $\hat{T} < \infty$ such that whenever $I \geqslant \hat{I}$ and $T \geqslant \hat{T}$, there is a sequential equilibrium with $(u_1, u_2, u_B) \in B_\varepsilon(u)$.*

Proof. A preliminary step is to construct equilibria of a very simple type. On the equilibrium path, the inefficient seller 2 sells at price v_2 in the first K periods and in the remaining $T - K$ periods, the efficient seller sells at price Δv. A deviation by any buyer (toward seller 1) in any of the first K periods results immediately in a switch to the sequential equilibrium minmaxing both sellers. The (average) cost of such a deviation for seller 1 is given by

$$\frac{I(T-K)(v_1 - v_2)}{T},$$

whereas the (average) gain for an individual buyer is bounded from above by

$$v_1 - \frac{T-K}{T} v_2.$$

Observe that we are counting the gains to a single buyer here. In sequential equilibrium, there are lots of degrees of freedom in the buyers' beliefs about the offers made to other buyers upon observing a deviation. In particular this allows all of the buyers to infer that whenever an individually acceptable deviating price is observed, that same price is offered to all other buyers. Since a single deviation by any buyer is sufficient to trigger the punishment for the firms, all buyers may expect that buyer $i^* \in \{1, ..., I\}$ must incur the cost of accepting the offer. As a result, the seller can expect to recoup the gains of at most a single buyer from any deviation. The cost to the seller equals the gain of a single buyer if

$$\frac{K}{T} = \frac{I v_1 - I v_2 - v_1 + v_2}{I v_1 - I v_2 + v_2},$$

and as a consequence, we can solve for the maximal number $K = K(T)$ that is consistent with an equilibrium of the form above.

Observe next that $K/T \to 1$ as $I \to \infty$, and as a consequence, we conclude that

$$(0, v_2, 0)$$

can be achieved as an average equilibrium payoff as $I \to \infty$. For any given I, we can support equilibrium average payoff (per buyer) vectors of the form:

$$\left(\frac{v_1(v_1 - v_2)}{Iv_1 - Iv_2 + v_2}, \left(1 - \frac{v_1}{Iv_1 - Iv_2 + v_2} \right) v_2, \frac{v_1 v_2}{Iv_1 - Iv_2 + v_2} \right). \qquad (2)$$

The next step is to use this equilibrium as a punishment for the buyers. Since the price of the seller who is not making sales in a given period can always be increased to the point where the buyer is indifferent between the two sellers, we can consider the action of rejecting both sellers as a deviation by the buyer. It is then possible to support average payoffs

$$\left(\frac{v_1(v_1 - v_2)}{Iv_1 - Iv_2 + v_2}, \left(1 - \frac{v_1}{Iv_1 - Iv_2 + v_2} \right) v_1, \frac{v_1 v_2}{Iv_1 - Iv_2 + v_2} \right)$$

in equilibrium. Notice that here the payoff for seller 2 converges to v_1 as $I \to \infty$.

The strategies supporting these payoffs are as follows. In the first $\sqrt{T}$ periods, seller 2 charges

$$\left(1 - \frac{v_1}{Iv_1 - Iv_2 + v_2} \right) v_1 \sqrt{T}$$

The buyers can be induced to purchase (even though it would be myopically better not to) by a threat of switching to the equilibrium leading to the payoffs in (2) if any buyer deviates. In the second phase of length

$$K - \sqrt{T},$$

the buyers purchase from seller 1 at price 0 to recoup the losses and in the third phase, approximately of length $T - K$, the equilibrium leading to payoffs (2) is played. The idea behind the new equilibrium is that the length of time at which seller 2 sells is minimized so that the efficiency losses are minimized, but seller 2 can extract a large part of the surplus by charging initially high prices.

Using similar arguments as before, it is again possible to show that all the other vertices of U^F can also be approximated given that the number of buyers is sufficiently large and that there are sufficiently many periods in the game. ∎

It should be remarked that the strategies employed in Proposition 2 are very sensitive to collusion among the buyers. By sharing the cost of a single deviation, the buyers can improve their payoffs dramatically in the game. Notice that we have not proved that the equilibria above support the maximal collusion in this game for finite I and T.

5. CONCLUSION

This note shows that restrictions on the observability of past prices in finitely repeated price competition games for a forward looking buyer have dramatic effects on the equilibrium outcomes. When prices are restricted to be non negative, and the game under perfect monitoring has a single subgame perfect equilibrium payoff vector, the set of sequential equilibrium payoffs in the game where the opponent's past prices are not observable is much larger. When prices are not restricted and there is full transferability of payoffs between each seller and the buyer (in the form of prices in each period), the subgame perfect equilibrium payoff set is the full set of feasible, individually rational payoffs in the game with perfect monitoring. In the game with private monitoring, however, the pure strategy sequential equilibrium payoff set coincides with the set of stage game subgame perfect equilibrium payoffs.

Potential extensions of the current model include finitely repeated auctions where the past bids are not made public and repeated contests in a principal agent framework.

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