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INCOMPLETE INSURANCE AND ABSOLUTE RISK AVERSION

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Suppose that a subset of states of nature are not verifiable individually. Given an optimal feasible insurance scheme, the expected utility across a group of unverifiable states is greater (less) than that of a verifiable state, if the degree of absolute risk aversion is decreasing (increasing).

1. Introduction

Suppose that a risk-averse agent faces different incomes for different states of nature. It is well-known that optimal risk-sharing between a risk-averse agent and a risk-neutral (insurance) agent results in the equalization of net income (income after premium and coverage) across different states of nature, given that all states of nature are verifiable. Suppose now that a subset of states of nature are not verifiable individually, but only as a group of states, by an insurance agent. Then the optimal insurance arrangement implies that the *marginal* utility of the net income at a verifiable state of nature is equal to the *expected marginal* utility of net income over a group of unverifiable states of nature. Given this optimality condition, we show that the (total) utility of a verifiable state is greater than, equal to, or less than the *expected* (total) utility of a

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group of unverifiable states depending on whether the degree of absolute risk aversion is decreasing, constant, or increasing, respectively. Before we prove the result in the third section, an economic example is given to illustrate the situation of incomplete insurance.

2. Economic example

Suppose that a firm announces to retain only a certain proportion, say ϕ, of its workers at random. The rest are laid off without pay from this firm. Each employee has an identical monotone increasing strictly-concave utility function, $u(y)$, where y is the net monetary income. A retained

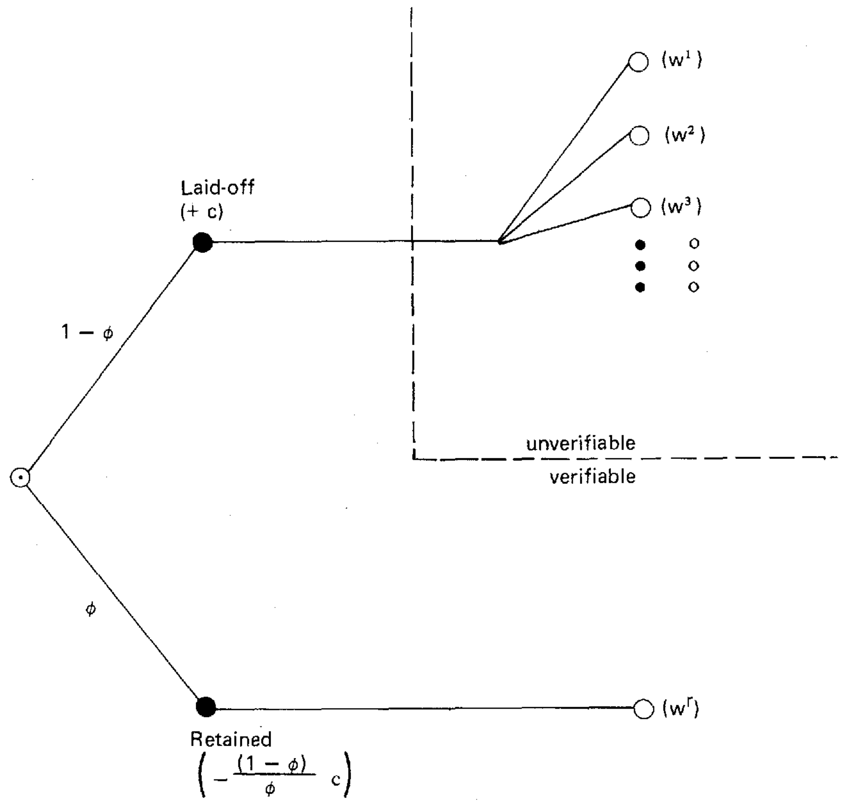

Fig. 1.

worker is paid, w^r. A laid-off worker may be employed in another firm with an uncertain rehiring wage, $\tilde{w} \geqslant 0$. The probability distribution function for the rehiring wage, $f(\tilde{w})$, is identical for and known to all workers. Let us assume

$$u(w^r) > \int_0^\infty u(\tilde{w}) \mathrm{d}\tilde{w} \equiv E_{\tilde{w}}(\tilde{w}),$$

where E is an expectation operator. Workers of a given firm can get together and arrange an unemployment insurance scheme by themselves to protect themselves against the risk of layoff. Once a worker leaves the original firm and finds a job at another firm, there are several problems in insuring his real income. First, there is an obvious moral hazard problem in search activity. Second, the precise verification of real income adjusted for fringe benefits and working conditions is impossible. Therefore, let us assume that an insurance policy can make payments conditional on whether a worker is laid off or retained, but not on the stochastic rehiring wage. A break-even insurance policy for layoffs implies that each laid-off worker receives c, while each retained worker pays in $(1-\phi)\, c/\phi$. (See fig. 1.) Expected utility with this scheme is

$$\phi u(w^r - (1-\phi)\, c/\phi) + (1-\phi)\, E_{\tilde{w}} u(\tilde{w} + c).$$

It is easy to verify that the optimal c is determined so that the following is true:

$$u'(w^r - (1-\phi)\, c/\phi) = E_{\tilde{w}} u'(\tilde{w} + c). \tag{1}$$

Eq. (1) implies that the *marginal* utility of a retained worker is equal to the *expected marginal* utility of a laid-off worker. We are now interested in whether the (total) utility of a retained worker is higher than, equal to, or lower than the expected (total) utility of a laid-off worker.

3. Mathematical question and answer

Now we have a general mathematical question. Introducing a symbol y for net income after premium and coverage, i.e., $\bar{y} \equiv w^r - (1-\phi)\, c/\phi$ and $\tilde{y} = \tilde{w} + c$, eq. (1) is rewritten as

$$u'(\bar{y}) = E_{\tilde{y}} u'(\tilde{y}), \tag{2}$$

where the probability distribution of $\tilde{y}$ is given and $\bar{y}$ is defined to satisfy eq. (2). Given (2), can we determine whether $u(\bar{y})$ is greater than, equal to, or less than $Eu(\tilde{y})$? The following theorem answers this question:

Theorem. Suppose that eq. (2) is given. Then

(i) $u(\bar{y}) > Eu(\tilde{y})$, *if u is a type of* increasing *absolute risk aversion*,
(ii) $u(\bar{y}) = Eu(\tilde{y})$, *if u is a type of* constant *absolute risk aversion*,
(iii) $u(\bar{y}) < Eu(\tilde{y})$, *if u is a type of* decreasing *absolute risk aversion*.

Proof. Define the inverse of the marginal utility function by Z:

$$Z \equiv (u')^{-1}.$$

Function Z is well-defined because u' is strictly monotone decreasing. Next, define v as the composite function of u and Z:

$$v \equiv u \circ Z.$$

Denoting by α a value of marginal utility of income, the following relation holds:

$$v(\alpha) = u(Z(\alpha)). \tag{3}$$

Differentiating both sides of (3) with respect to α to obtain

$$v'(\alpha) = u'(Z(\alpha)) \cdot Z'(\alpha).$$

By the inverse function theorem $Z'(\alpha) = 1/u''(Z(\alpha))$. Hence, noting $Z(\alpha) = y$, and $\alpha = u'(y)$,

$$v'(u'(y)) = u'(y)/U''(y). \tag{4}$$

Let us introduce the degree of absolute risk aversion (hereafter, A.R.A.) in the sense of Pratt (1964) and Arrow (1965),

$$\rho(y) = -u''(y)/u'(y).$$

Then $v'(u'(y)) = -1/\rho(y)$. Increasing (decreasing; constant) absolute risk aversion implies $\rho'(y) > 0$ $(<0; =0)$. Now we take the second derivative of v,

$$\begin{aligned} v''(\alpha) &= (\mathrm{d}/\mathrm{d}\alpha)(-1/\rho) \\ &= (\mathrm{d}/\mathrm{d}y)(-1/\rho)\cdot \mathrm{d}y/\mathrm{d}\alpha \\ &= (\mathrm{d}/\mathrm{d}y)\left(-\rho(y)^{-1}\right)/u''(y) \\ &= \rho'(y)/(\rho(y))^2\cdot u''(y). \end{aligned} \tag{5}$$

Recalling $u''(y)<0$, eq. (5) implies

$v''(\alpha)>0,$	iff	$\rho'(y)<0,$	i.e., decreasing A.R.A.,
$v''(\alpha)=0,$	iff	$\rho'(y)=0$	i.e., constant A.R.A.,
$v''(\alpha)<0,$	iff	$\rho'(y)>0,$	i.e., increasing A.R.A.

Let us consider a pair of marginal and total utility corresponding to an arbitrary income level, $(u'(y), v(u'(y)))$, or $(\alpha, v(\alpha))$. Let us take the case of $v''>0$. Jensen's inequality yields

$$v(E_{\tilde{y}}u'(\tilde{y}))<E_{\tilde{y}}(v(u'(\tilde{y}))). \tag{6}$$

The condition (2) defines unique $\bar{y}$:

$$u'(\bar{y})=E_{\tilde{y}}u'(\tilde{y}). \tag{2}$$

The existence of $\bar{y}$ is guaranteed by continuity and uniqueness is by $u''<0$. Then the left-hand side of (6) becomes

$$v(E_{\tilde{y}}(u'(\tilde{y})))=u(Z(E_{\tilde{y}}u'(\tilde{y})))=u(Z(u'(\bar{y})))=u(\bar{y}).$$

The right-hand side of (6) is

$$E_{\tilde{y}}(v(u'(\tilde{y})))=E_{\tilde{y}}(u(\tilde{y})).$$

Hence, (6) implies

$$u(\bar{y})<E_{\tilde{y}}u(\tilde{y}).$$

Similarly, if $v''<0$, then

$$v(E_{\tilde{y}}u'(\tilde{y}))>E_{\tilde{y}}(v(u'(y))),$$

which results in

$$u(\bar{y}) > E_{\tilde{y}} u(\tilde{y}).$$

The case of $v'' = 0$ is proved by replacing inequalities above by equalities Q.E.D.

4. Some remarks on an implication of theorem

If the degree of absolute risk aversion is decreasing, then the theorem in section 3 has a counter-intuitive implication for the economic example in section 2. Given an optimal insurance scheme against layoffs alone, a laid-off worker who does not yet know the rehiring wage elsewhere enjoys a higher level of expected utility than a retained worker. Therefore, every worker wants to be laid off. In order to prevent a worker from voluntarily quitting, the payment of c to the laid-off worker should not be paid in the event of a voluntary quit. The same problem will arise in the context of implicit contract theory if there are severance payments. The severance payments should not be paid in the event of a voluntary quit in order that optimal implicit contracts are implementable. [See Geanakoplos and Ito (1981) for this point.]

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