Cowles Foundation Paper 600

Journal of Mathematical Economics 13 (1984) 1–9 North-Holland

UTILITY FUNCTIONS FOR DEBREU'S 'EXCESS DEMANDS'

John GEANAKOPLOS*

Yale University, New Haven, CT 06520, USA

Received January 1979, final version accepted September 1983

Given an arbitrary function $x: \mathbb{R}^l \to \mathbb{R}^l$ satisfying Walras law and homogeneity, Debreu decomposed x into the sum of l 'individually rational' functions $x(p) = \sum_{k=1}^{l} \bar{x}^k(p)$. Here we find explicit utility functions u^k, constructed on the basis of a simple geometric intuition, which give rise to Debreu's excess demands $\bar{x}^k(p)$.

In a series of papers by Sonnenschein (1973), Mantel (1974), Debreu (1974), McFadden et al. (1974) and Mantel (1976), it has been demonstrated that neoclassical microeconomic theory imposes almost no restriction on community excess demand functions other than Walras law and homogeneity, if the economy contains no more commodities than consumers. Unfortunately, many of the ideas in these important proofs are hidden by the extremely complicated nature of the constructions.

Perhaps the most remarkable of these proofs is Debreu's. He decomposes an arbitrary continuous function $x(p)$ on $\mathring{R}^l_+$ (the 'candidate excess demand') satisfying Walras law and homogeneity of degree zero into l functions $\bar{x}^k(p)$, $k=1,\dots,l$ and he constructs l systems of convex, monotonic indifference curves in such a way that, subject to the budget constraint $p^t x \leqq 0, \bar{x}^k(p)$ lies on the highest indifference curve of the kth system, $k=1,\dots,l$, (so long as p is not too close to the boundary where some price is zero). In this paper I write down explicit concave and monotonic utility functions $u^k, k=1,\dots,l$, constructed on the basis of a simple geometric intuition, such that maximizing the kth utility function, u^k, subject to the budget constraint $p^t x \leqq 0$, gives exactly the kth individual excess demand $\bar{x}^k(p)$ in Debreu's decomposition (away from the boundary). In order to guarantee concavity and monotonicity of the utility functions I impose the restriction that the original $x(p)$ be differentiable as well as continuous, but I hope thereby to bring the ideas lying behind Debreu's decomposition into sharper focus.

The plan of this article is as follows. First the arbitrary $x(p)$ is decomposed, $x(p) = \sum_{k=1}^{l} \bar{x}^k(p) = \sum_{k=1}^{l} \beta_k(p) \hat{x}^k(p)$ as in the Debreu paper, where only the scalar functions β_k depend on the original candidate excess demand $x(p)$.

*I would like to acknowledge helpful conversations with Heraklis Polemarchakis, Truman Bewley and Bob Anderson, as well as the very useful comments of an anonymous referee

In Part 2 we observe that $\hat{x}^k(p)$ is the closest point in the budget set $\{x \in \mathbb{R}^l \mid p^t x \leqq 0\}$ to the kth unit vector e^k. We also note that $\hat{x}^k(p)$ and $\bar{x}^k(p)$ are elements of $\tilde{X}^k = \{x \in \mathbb{R}^l \mid x_i < 0, i \neq k, x_k > 0\}$ for all $p \in \mathring{\mathbb{R}}^l_+$, and conversely that for any $x \in \tilde{X}^k$ there is a unique $p(x)$, namely $p(x) = e^k - x_k x/|x|^2$, such that $\hat{x}^k(p(x))$ and $\bar{x}^k(p(x))$ are scalar multiples of x. It follows immediately that $p(\bar{x}^k(p)) = p$ and hence that the utility function u^k defined on all of $\tilde{X}^k$ by $u^k(x) = -(|\hat{x}^k(p(x)) = e^k|^2 + |x - \bar{x}^k(p(x))|^2)$ is uniquely maximized over the set $\{x \in \mathbb{R}^l \mid p^t x \leqq 0\} \cap \tilde{X}^k$ at $\bar{x}^k(p)$, for any $p \in \mathring{\mathbb{R}}^l_+$.

In order to assure monotonicity and concavity, we must restrict our attention in Part 3 to a compact set of prices P, for instance $\{p \in R^l \mid P_i \geqq \varepsilon, i = 1, \ldots, l, |p| = 1\}$ for any $\varepsilon > 0$. Then each individual excess demand $\bar{x}^k(p)$ will lie in some large convex and compact subset X^k of R^l. By perturbing the utilities and taking advantage of the differentiability assumption, it is shown that each utility u^k can be taken to be monotonic and strictly concave on X^k and still give rise to the same $\bar{x}^k(p)$. Finally, in Part 4 of the argument, u^k is extended to all of R^l, preserving concavity, monotonicity and the excess demands for all $p \in P$, but perhaps not for p outside of P. Observe that since X^k is bounded, we can choose $w^k \in R^l_+$, the initial endowment vectors for $k = 1, \ldots, l$, large enough so that the net trade space $R^l_+ - w^k$ contains X^k for all $k = 1, \ldots, l$.

Notation

Let $\mathring{R}^l_+ = \{p \in R^l \mid p_i > 0, i = 1, \ldots, l\}$. We denote by $T(p)$ the set $\{x \in R^l \mid p^t x = 0\}$ for all $p \in \mathring{R}^l_+$, where $p^t x$ means p transpose. We let $p \perp x$ mean $p^t x = 0$.

Let e^k be the kth standard basis vector, $k = 1, \ldots, l$ and let $\Pi_{T(p)} y$ be the projection of y perpendicularly onto $T(p)$, i.e., in the direction p. Then $\Pi_{T(p)} y = [I - pp^t/|p|^2] y$.

We write $x > y$ iff $x_i \geqq y_i, i = 1, \ldots, l$ and $x \neq y$ and $x \gg y$ iff $x_i > y_i, i = 1, \ldots, l$. Let $x(p)$ denote a function $x: \mathring{R}^l_+ \to R^l$. We call $x(p)$ a *candidate aggregate excess demand* function if it satisfies:

(1) Homogeneity (H): $x(p) = x(\lambda p)$ for all $\lambda > 0$ and all $p \in \mathring{R}^l_+$.
(2) Walras law (W): $p^t x(p) = 0$ for all $p \in \mathring{R}^l_+$.
(3) Twice continuous differentiability (C^2): $x(p)$ is C^2 on $\mathring{R}^l_+$.

We define a rational agent as an ordered pair (u, X) satisfying:

(1) $X = R^l_+ - w$, for some $w \in R^l_+$, that is X is the set of net trades.
(2) u is a function u: $X \to R$ which is monotonic, $x > y \Rightarrow u(x) > u(y)$ and concave, $u(\lambda x + (1-\lambda) y) \geqq \lambda u(x) + (1-\lambda) u(y)$ for all x and y and $0 \leqq \lambda \leqq 1$.
(3) Given $p \in \mathring{R}^l_+$, the agent always acts to maximize $u(x)$ such that $x \in X, p^t x \leqq 0$. If $x(p)$ is the unique solution to a rational agent's maximization problem for all $p \in P \subset \mathring{\mathbb{R}}_+$, and if $x(p)$ is C^2, then we call $x(p)$ a *rational individual excess demand* on P.

Recall that u is monotonic if $Du(x) \gg 0$ and strictly quasi-concave if $y'D^2u(x)y < 0$ for all $y \perp Du(x)$ for all $x \in X$.

Theorem. *Let $x(p)$ be a candidate excess demand function, $x: \mathring{R}^l_+ \to R^l$, and let P be a compact set, $P \subset \mathring{R}^l_+$. Then there exist l rational agents $\{(u^k, X^k), k=1,\ldots,l\}$ giving rise to l rational individual excess demands $\bar{x}^k(p)$ such that $\sum_{k=1}^l \bar{x}^k(p) = x(p)$ for all $p \in P \subset \mathring{\mathbb{R}}^l_+$.*

Part 1. Decomposition. By Walras law, $p^t x(p) = 0$ for all $p \in \mathring{R}^l_+$, i.e., $x(p) \in T(p)$. Now for all $p \in \mathring{R}^l_+$, we can find a scalar $\theta(p)$ such that $x(p) + \theta(p)(p/\|p\|) \gg 0$. If $x(p)$ is C^2 and homogeneous, then θ can be chosen so as well.

Let $\Pi_{T(p)}y$ denote the projection of y perpendicularly onto $T(p)$. (See fig. 1). If $\{e^1, e^2, \ldots, e^l\}$ is the standard basis for $\mathbb{R}^l$ and $x(p) = (x_1(p), \ldots, x_l(p))^t =$

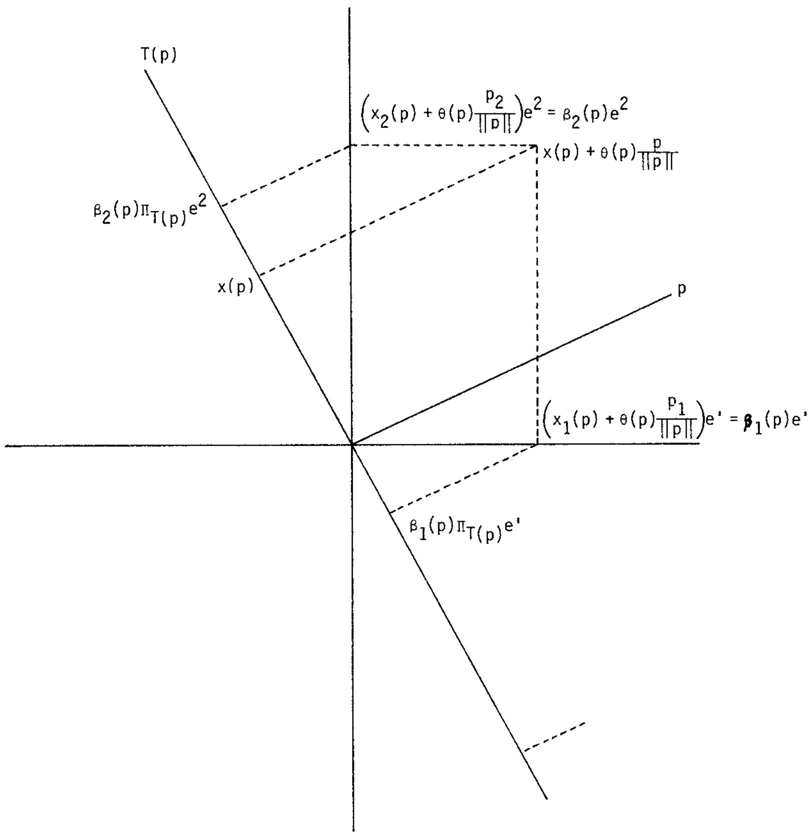

Fig 1

$\sum_{k=1}^{l} x_k(p)e^k$, then by noting that $\Pi_{T(p)}\theta(p)(p/\|p\|)=0$, we can set

$$x(p)=\Pi_{T(p)}x(p)=\Pi_{T(p)}\left[x(p)+\theta(p)\frac{p}{\|p\|}\right]$$

$$=\Pi_{T(p)}\left[\sum_{k=1}^{l}\left(x_k(p)+\theta(p)\frac{p_k}{\|p\|}\right)e^k\right]$$

$$=\sum_{k=1}^{l}\left(x_k(p)+\theta(p)\frac{p_k}{\|p\|}\right)\Pi_{T(p)}e^k.$$

Let $\beta_k(p)=x_k(p)+\theta(p)(p_k/\|p\|)$ and $\bar{x}^k(p)=\beta_k(p)\Pi_{T(p)}e^k$, $k=1,\dots,l$. We have just shown that $x(p)$ can be decomposed at all $p\in\mathring{\mathbb{R}}^l_+$ into l functions $\bar{x}^k(p)=\beta_k(p)\Pi_{T(p)}e^k$, $k=1,\dots,l$ satisfying H, W, and C^2 and that $\beta_k(p)>0$ for all $p\in\mathring{\mathbb{R}}^l_+$, and $k=1,\dots,l$.

$$x(p)=\sum_{k=1}^{l}\bar{x}^k(p)=\sum_{k=1}^{l}\beta_k(p)\Pi_{T(p)}e^k.$$

Observe that $\Pi_{T(p)}e^k$ is a vector with a strictly positive kth coordinate and $l-1$ strictly negative components, that is, $x_i^k(p)<0, i\neq k$ and $0<x_k^k(p)$ for $k=1,\dots,l$. This will be important later.

Note that so far the only use of the continuity of $x(p)$ was to show that the $\beta_k(p)$ are continuous. We will not need continuity to construct our utility functions (the construction is entirely geometrical). We need it to prove continuity of the utility functions and we need continuous differentiability to prove that the gradients of u^k exist and are monotonic and bounded away from zero on a compact set and twice continuous differentiability to prove concavity of the utility functions.

Part 2. There exist functions $u^1,\dots,u^l$ such that $\max[u^k(x)$ such that $x\in\{x\in\tilde{X}^k\mid p^t x\leqq 0\}]$ is uniquely attained at $\bar{x}^k(p)$ for all $p\in\mathring{R}^l_+$ and $k=1,\dots,l$.

As a first step, consider the special case where $\beta_k(p)=1$ for all p. Then $\bar{x}^k(p)=\Pi_{T(p)}e^k$.

Lemma 1. Let $\tilde{u}_y(x)=-\|x-y\|^2=-\sum_{k=1}^{l}(x_k-y_k)^2$. Then $\tilde{u}_y$ is a monotonic strictly concave utility function on $X=\{x\in\mathbb{R}^l\mid x_i<y_i, i=1,\dots,l\}$ whose derived rational individual excess demand $z(p)$ is exactly $\Pi_{T(p)}y$ for all $p\in\mathbb{R}^l$ with $p^t y>0$.

Proof. $u_y(x)$ is exactly the negative of the square of the distance between x and y. Maximizing $\tilde{u}_y(x)$ on $\{x\in\mathbb{R}^l\mid p^t x\leqq 0\}\equiv B(p)$ is equivalent to minimizing the distance between y and $B(p)$, which obviously occurs uniquely at $\Pi_{T(p)}y$ so long as $p^t y \geqq 0$. See fig. 2. Moreover, on $X, \tilde{u}_y$ is differentiable and $D\tilde{u}_y(x) = D[-\sum_{i=1}^{l}(x_i - y_i)^2] = -2(x_1 - y_1, \ldots, x_l - y_l) \gg 0$ since $x \ll y$, hence $\tilde{u}_y$ is monotonic. Furthermore, $D^2\tilde{u}_y(x) = -2I$, hence $\tilde{u}_y$ is also strictly concave. Q.E.D.

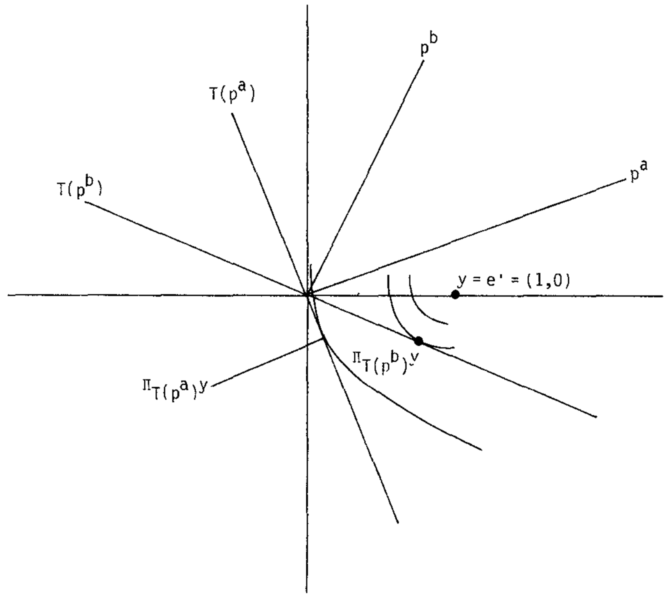

Fig. 2

Since $p^t e^k > 0$ for all $p \in \mathring{R}^l_+$ and $k = 1, \ldots, l$, it follows that $\hat{x}^k(p) \equiv \Pi_{T(p)}e^k$ is a rational individual excess demand function that can be derived from the utility function $\tilde{u}^k(x) = -\|x - e^k\|^2$, $k = 1, \ldots, l$, once $\tilde{u}^k$ is properly extended beyond X.

The difficulty that remains is to prove that we can modify $\tilde{u}^k(x)$, getting $u^k(x)$ such that the solution to $\max u^k(x)$ given $x \in \{x \in \tilde{X}^k | p^t x \leqq 0\}$ is $\bar{x}^k(p) = \beta_k(p)\Pi_{T(p)}e^k$ rather than $\Pi_{T(p)}e^k$.

The Debreu construction of the indifference curves can be thought of as the bending of the circles centered at e^k until they are tangent to $T(p)$ at $\beta_k(p)\Pi_{T(p)}e^k$ instead of at $\Pi_{T(p)}e^k$. See fig. 3. Debreu proved that this is possible on $\mathbb{R}^l_\varepsilon = \{p \in \mathring{\mathbb{R}}^l_+ \mid (p_i/\|p\|) \geqq \varepsilon\}$. We shall show it is actually possible on all of $\mathring{\mathbb{R}}^l$ by defining a utility function on all $\tilde{X}^k = \{x \in \mathbb{R}^l \mid x_i < 0, i \neq k, x_k > 0\}$. The idea is that given any $x \in \tilde{X}^k$ we can find a unique price vector $p(x) \in \mathring{\mathbb{R}}^l_+$ and a unique multiple $\lambda(x)x$ of x such that $\lambda(x)x = \Pi_{T(p(x))}e^k$. From Pythagoras' law we have with this notation $\tilde{u}_k(x) = -\|x - e^k\|^2 = -(\|\Pi_{T(p(x))}e^k - e^k\|^2 + \|x - \Pi_{T(p(x))}e^k\|^2) = (\|\lambda(x)x - e^k\|^2 + \|x - \lambda(x)x\|^2)$. We can get the correct excess demand by defining $u^k(x) = -(\|\Pi_{T(p(x))}e^k - e^k\|^2 + \|x - \beta_k(p(x))\Pi_{T(p(x))}e^k\|^2)$.

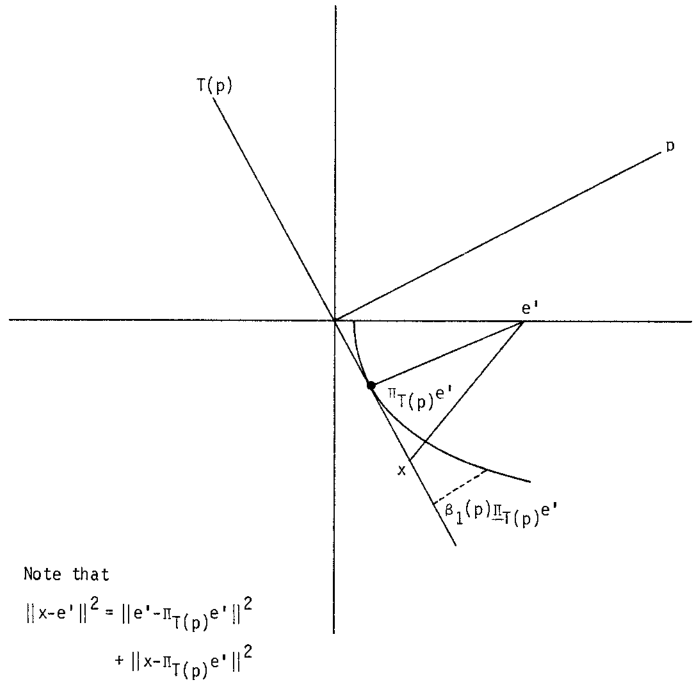

Fig. 3

Proof of Part 2. The ray $\{\lambda x \mid \lambda > 0\}$ and the point e^k not on the line (since $\lambda x_i < 0$ and $e_i^k = 0$ for $i \neq k$) determine a plane. Hence there is a unique line perpendicular to x through e^k, namely $p(x) = e^k - x_k(x/|x|^2)$. Observe that $p(x)^t x = x_k - x_k(|x|^2/|x|^2) = 0$ and that, letting $\lambda(x) = x_k/|x|^2$, $\lambda(x)x + p(x) = e^k$, hence indeed $\Pi_{T(p(x))} e^k = \lambda(x)x$. See fig. 4. It can be seen that the vector p is the vector of residuals derived from the regression of e^k onto x and that the first term in the above expression is simply the mean squared error of that regression, namely

$$-\left\|\frac{x^t e^k}{|x|^2} x - e^k\right\|^2 = -\left\|\frac{x_k x}{|x|^2} - e^k\right\|^2 = -\left(\frac{x_k^2}{|x|^2} - 2\frac{x_k^2}{|x|^2} + 1\right) = \frac{x_k^2}{|x|^2} - 1.$$

Geometrically, as p varies through $\mathring{R}^l_+$, $\Pi_{T(p)} e^k$ traces out the hemisphere with center at $\frac{1}{2}e^k$. Given any $x \in \tilde{X}^k$, the line from the origin through x intersects that hemisphere at exactly one point, $\lambda(x)x$. The line segment from $\lambda(x)x$ to e^k is $p(x)$. Recall from elementary geometry that two lines connecting the two endpoints of the diameter of a (semi)circle to the same point on the semicircle must meet at a right angle. Observe that p is uniquely determined up to positive scalar multiples by the line x and the point e^k.

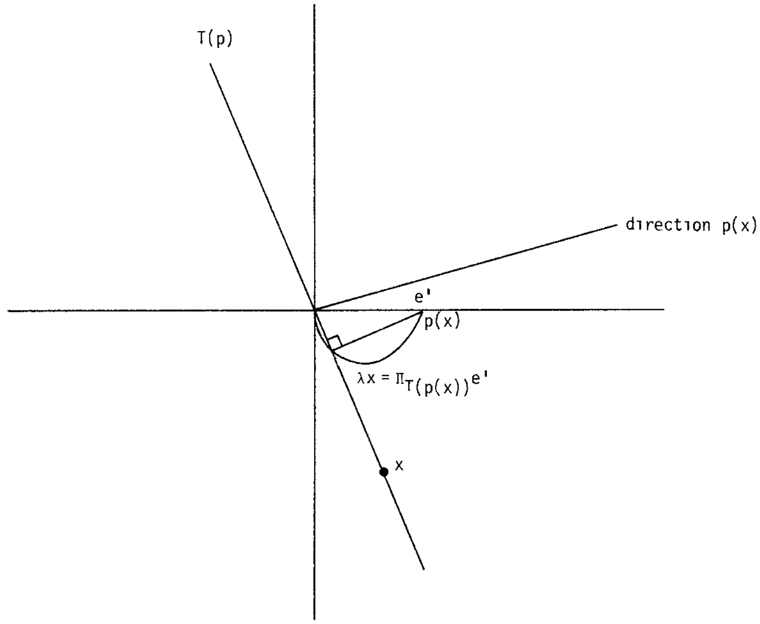

Fig. 4

Note that in fact p is a differentiable function of x. Note that $\tilde{X}^k$ was chosen so that the uniquely defined $p(x)$ is strictly positive for all $x \in \tilde{X}^k$. See fig. 4.

Observe that if $x = \Pi_{T(\bar{p})} e^k$, then $p(x)$ is indeed $\bar{p}$, and if $x = \beta \Pi_{T(\bar{p})} e^k$, then also $p(x) = \bar{p}$. Thus we can define

$$u^k(x) = -\|\Pi_{T(p(x))} e^k - e^k\|^2 - \|x - \beta_k(p(x)) \Pi_{T(p(x))} e^k\|^2.$$

From the above demonstration we know that the problem $\max u^k(x)$ such that $x \in \tilde{X}^k$ and $p^t x \leqq 0$ is solved uniquely by $\beta_k(p) \Pi_{T(p)} e^k = \bar{x}^k(p)$ since $p(\bar{x}^k(p)) = p$ and any multiple of $\Pi_{T(p)} e^k$ maximizes the first term of u^k subject to the budget constraint and $\bar{x}^k(p)$ uniquely maximizes the second term by making it zero. Q.E.D.

Part 3. Now let

$$u^k(x) = -\|\Pi_{T(p(x))} e^k - e^k\|^2 - \varepsilon \exp\{n \|x - \beta_k(p(x)) \Pi_{T(p(x))} e^k\|\}.$$

For ε small enough and n big enough, this is monotonic and strictly quasi-concave on a compact, convex set X^k containing $\{\bar{x}^k(p) \mid p \in P\}$.

Proof of Part 3. Notice first that the derived demands are unaffected. Now suppose P is compact in $\mathring{R}^l_+$; then $\{\bar{x}^k(p)|p\in P\}$ is compact if $\bar{x}^k$ is C^0, hence we can find a closed convex, bounded X^k such that $\{\bar{x}^k(p)|p\in P\}\subset X^k\subset \tilde{X}^k$. Then if x is C^1, so that $\beta_k(p)$ is C^1, the gradient of $\|x-\beta_k(p(x))\Pi_{T(p(x))}e^k\|^2$ is bounded from below on X^k. But it is obvious that the gradient of $-\|\Pi_{T(p(x))}e^k-e^k\|^2$ is proportional to $p(x)\gg 0$ and hence is bounded from below by a strictly *positive* constant in every coordinate on X^k. Hence for all sufficiently small $\varepsilon>0$, $u^k(x)$ is monotonic on X^k and gives rise to the excess demand $\bar{x}^k(p)$.

The first term, $v(x)=-\|\Pi_{T(p(x))}e^k-e^k\|^2=(x_k^2/|x|^2)-1$ is concave in the remaining $l-1$ variables given any fixed value of x_k and therefore the second derivative D^2v is negative definite on $[e^k]^\perp$. Since $e^k\in[p(x),x]$ it follows that D_v^2 is negative definite on $[p(x),x]^\perp$. It is clearly identically 0 in the direction x. Furthermore, let $f(x)=\|x-\beta(p(x))\Pi_{T(p(x))}e^k\|$. Since $p(x)=p(\lambda x)$ for all $\lambda>0$, it is easy to see that $D^2f^2(x)$ and also $D^2w_n(x)=D^2[-e^{nf(x)}]$ are negative definite in the direction x. Thus for small enough ε we know that $D^2u^k(x)=D^2v(x)+\varepsilon D^2w_n(x)$ is negative definite on $[p(x)]^\perp$. In order to prove the quasi-concavity of u^k on X^k, we must show that D^2u^k is negative definite on $[Du^k(x)]^\perp=[Dv(x)+\varepsilon Dw_n(x)]^\perp=[\lambda p(x)+\varepsilon Dw_n(x)]^\perp$.

Any $y\in S^{l-1}$ can be uniquely written as $y=\alpha_1(p(x)/|p(x)|)+\alpha_2(x/|x|)+\alpha_3(q_3/|q_3|)+\cdots+\alpha_n(q_n/|q_n|)$, where $q_i\in[p(x),x]^\perp$, $i=3,\ldots,n$. Moreover, if $y\perp Du^k(x)$, then $|\alpha_1|\leqq M\varepsilon|Dw_n|$, where the number M can be taken to be the same for all x in the compact set X^k. Imagine expressing the matrix $D^2u^k=D^2v+\varepsilon D^2w_n$ in the orthogonal basis

$$\left(\frac{p(x)}{|p(x)|},\frac{x}{|x|},\frac{q_3}{|q_3|},\ldots,\frac{q_n}{|q_n|}\right)$$

and then calculating $y^tD^2u^ky$. If $\alpha_1=0$, then we would be done. If $\alpha_2=0$, then for small enough ε we would also get $y^tD^2u^ky<0$, since D^2v is negative definite on $[p(x),x]^\perp$ and $\alpha_1\to 0$ and $\varepsilon D^2w_n\to 0$ as $\varepsilon\to 0$. The only difficult case arises when $\alpha_3=\cdots=\alpha_n=0$. Then $y^tDu^ky=\alpha_1^2v_{pp}+2\alpha_1\alpha_2v_{px}+\alpha_2^2\cdot 0+\alpha_1^2\varepsilon w_{pp}^n+2\alpha_1\alpha_2\varepsilon w_{px}^n+\alpha_2^2\varepsilon w_{xx}^n$. Now notice that as $\varepsilon\to 0$, so that $\alpha_1\to 0$, all the terms except $2\alpha_1\alpha_2v_{px}+\alpha_2^2\varepsilon w_{xx}^n$ are $0(\varepsilon^2)$ and effectively disappear. We can assume that $\alpha_2^2=1-\alpha_1^2$, that $|\alpha_1|\leqq M\varepsilon|Dw_n|$, and that $w_{xx}^n<0$. It follows that if $|w_{xx}^n|/|Dw_n|$ is sufficiently big, then this last remaining sum is negative. And that is the point of introducing the exponential operator here. The reader can quickly verify that by taking n large, the ratio of second derivative to first derivative of e^{-nx} can be made arbitrarily large. It follows similarly that

$$\frac{|w_{xx}^n|}{|Dw_n|}=\frac{n}{|Df(x)|}$$

can be made arbitrarily large on the compact set X^k by taking n sufficiently large.

Part 4. We can modify the utility functions u^k one more time so that they are strictly concave and monotonic on X^k, $k=1,\dots,l$, and give rise to the same excess demands $\bar{x}^k(p)$ for $p \in P$. We can then extend the u^k to all of R^l, preserving monotonicity and concavity. The $\bar{x}^k(p)$ will again maximize u^k subject to the budget constraint $p^t x \leqq 0$, for all $p \in P$, $k=1,\dots,l$.

Proof of Part 4. Fortunately the proof of Part 4 is quite easy. Aumann (1975) demonstrated that it is always possible to monotonically transform a strictly quasi-concave C^2 function into a strictly concave function on any compact set X^k. Simply define $V^k(x) = -e^{-Nu^k(x)}$ for N big enough. Then $DV^k(x) = NDu^k e^{-Nu^k}$ and $D^2V^k = [ND^2u^k - N^2 Du^k(Du^k)^t]e^{-Nu^k}$ which is negative definite on $[Du^k]^\perp$ since D^2u^k is, and for N big enough is clearly negative definite on all of X^k.

To extend V^k to all of R^l, define $\bar{u}^k$ as the infimum of all linear functions that lie above V^k on all of X^k. Formally, for every $y \in X^k$, let Ly be the linear function $Ly(x) = V^k(y) + DV^k(y)^t(x-y)$. Since V^k is C^2 and concave on X^k, $Ly(x) \geqq V^k(x)$ for all $x \in X^k$ and $Ly(y) = V^k(y)$. Let $\bar{u}^k(x) = \inf\{Ly(x) \mid y \in X^k\}$. Since the inf of concave (linear) functions is concave, $\bar{u}^k$ is concave. Moreover, $\bar{u}^k(x) = V^k(x)$ for all $x \in X^k$. Furthermore, since X^k is compact and V^k continuously differentiable, $\bar{u}^k$ is well-defined and finite on all R^l. Q.E.D.

Finally, observe that it is possible to choose $w^k \in R^l_+$ such that $w^k \gg \bar{x}^k(p)$ for all $p \in P$. In that case we can more traditionally restrict the feasible net trade space to $R^l_+ - w_k$ without disturbing the maximization of utility for $p \in P$. If we had begun with an observable aggregate endowment w as well as $x(p)$, this last argument would not in general be valid. There would indeed be restrictions on community excess demands [at least $x(p) \geqq w$ for all p].

References

Aumann, R , 1975, Values of market games with a continuum of traders, Econometrica

Debreu, G , 1974, Excess demand functions, Journal of Mathematical Economics 1, 15–21.

McFadden, Mas-Collel, Mantel and Richter, 1974, A characterization of community excess demand functions, Journal of Economic Theory 9, 361–374

Mantel, R.R , 1974, On the characterization of aggregate excess demands, Journal of Economic Theory 7, 348–353

Mantel, R.R., 1976, Homothetic preferences and community excess demand functions, Journal of Economic Theory 12, 197–201

Sonnenschein, H , 1973, Do Walras identity and continuity characterize the class of community excess demand functions?, Journal of Economic Theory 6, 345–354