Reprinted from JOURNAL OF ECONOMIC THEORY
All Rights Reserved by Academic Press, New York and London
Vol 34, No 2, December 1984
*Printed in Belgium*

# Intertemporally Separable, Overlapping-Generations Economies*

J. D. GEANAKOPLOS

*Cowles Foundation, Yale University, New Haven, Connecticut 06520*

AND

H. M. POLEMARCHAKIS

*Graduate School of Business, Columbia University, New York, New York 10027*

Received May 2, 1983; revised March 27, 1984

Intertemporally separable, overlapping-generations economies typically have a sequentially finite number of equilibria, though the equilibrium set may, even with probability 1, be uncountable. *Journal of Economic Literature* Classification Numbers: 021, 022, 023. © 1984 Academic Press, Inc

## I. INTRODUCTION

A complete characterization of the set of equilibria of a typical overlapping-generations (OG) economy is still lacking. In particular, we have, so far, no satisfactory way of counting equilibria or of defining typical economies. Recently, Balasko and Shell [2] demonstrated that, when all but a finite number of the generations in an OG economy consist of a single consumer with Cobb–Douglas preferences, the number of equilibria is generically finite; when every generation has such a simple structure, the equilibrium is unique.

In this paper, we extend the Balasko–Shell argument, and we provide a complete characterization of the set of equilibria of any intertemporally separable, overlapping-generations (ISOG) economy; that is, any OG economy with two-period lives in which some infinite collection of

* This paper was prepared while the first author was visiting the University of Pennsylvania, where he had many helpful conversations with Karl Shell; he should also like to thank Don Brown for helpful comments. Both authors have received support from National Science Foundation grant SES 82-10034. Earlier versions of this paper appeared as D.P. No. 8342, CORE, Université Catholique de Louvain, and CFDP No. 667, Cowles Foundation, Yale University.



0022-0531/84 $3.00

generations each consists of a single agent with an intertemporally separable utility function

$$u_t(x_t^t, x_t^{t+1}) = f_t(u_t^t(x_t^t), u_t^{t+1}(x_t^{t+1}))$$
$$= f_t(u_t^t(x_t^{t,1},..., x_t^{t,l}), u_t^{t+1}(t_t^{t+1,1},..., x_t^{t+1,l})).$$

We use this characterization to define a regular ISOG economy and we show that, for regular ISOG economies, the number of equilibria can be described as sequentially finite; that is, for any $T$, the number of sequences $\bar{p}(T) = (\bar{p}^1,..., \bar{p}^T)$ which can be extended to equilibria $\bar{p} = (\bar{p}^1,..., \bar{p}^T, \bar{p}^{T+1},...)$ is finite. In particular, a small perturbation of an equilibrium price $\bar{p}_t$, $1 \leqslant t \leqslant \bar{T}$, will destroy the equilibrium, no matter how the other prices are changed to compensate. When every agent $(t, i)$ in a regular ISOG economy has utility function $u_{t,i}$ which gives rise to conventional gross substitute demands in $2l$ commodities (in particular, if there is one Cobb–Douglas consumer per generation), then the equilibrium is unique. However, we also note that, for more general regular ISOG economies, the number of equilibria, though sequentially finite, may nevertheless be uncountable, even if we restrict ourselves to Pareto optimal allocations.

We make precise the notion of a typical ISOG economy by arguing that, for fixed utility functions, the appropriate topology for the infinite dimensional space $\mathscr{E}$ of ISOG economies parametrized by endowments is neither the product topology nor the sup-norm topology, but rather what is called in global analysis the box, or fine, or Whitney topology.[1] The class of regular economies is shown to be typical (or generic) in the sense that it is an open and dense set in this fine topology and also of measure 1 with respect to the natural infinite product of Lebesgue measure. Thus, our main result can be simply stated: ISOG economies typically have a sequentially finite number of equilibria, though the equilibrium set may, even with probability 1, be uncountable.

Our proposition about the number of equilibria under separability was already established in Geanakoplos and Brown [5], where it was shown that the set of equilibria generically formed a 0-dimensional, non-standard manifold and hence contained a *-finite number of equilibria.[2] The crucial

[1] In Balasko and Shell [2], the space of economies could be parametrized by the finite-dimensional set of endowments of the non-Cobb–Douglas agents; thus no question of the appropriate topology arose.

[2] In a paper bearing the same title as this one, Kehoe and Levine [6], following precisely the idea in Geanakoplos and Brown [5], showed that the stable manifold of any regular steady-state is 0-dimensional. Here, as in Geanakoplos and Brown [5], we deal with economies which may differ from generation to generation and hence need not possess a steady-state; furthermore, we consider the entire set of equilibria, not just those converging to a steady-state.

idea needed there is the fact that, for $u_t$ separable, the matrix $M = D_{p^{t+1}} x_t^t(p^t, p^{t+1})$ of derivatives of the period $t$ demand with respect to period $t+1$ prices of the agent born at $t$ has rank 1. Here, we obtain the same conclusion by making an elementary observation about the budget constraint at equilibrium and by appealing to conventional generic properties of finite economies.

We offer our simple result both as an explanation of the Balasko–Shell example and as a guide or limitation to what might typically true, at least for fixed utilities, as endowments vary in more general OG economies.

## II. The Economy

Each agent $(t, i)$, $i \in I_t$, a finite set, $t = 1, \ldots$, is characterized by a strictly positive endowment vector $e_{t,i} = (e_{t,i}^t, e_{t,i}^{t+1}) \in \mathbb{R}_{++}^l \times \mathbb{R}_{++}^l$ and a twice continuously differentiable, strictly differentiably quasi-concave, and strictly differentiably monotonically increasing utility function $u_{t,i}: \mathbb{R}_{++}^l \times \mathbb{R}_{++}^l \to \mathbb{R}$. Furthermore, each agent $(0, i)$, $i \in I_0$, a finite set, is characterized by a strictly positive endowment vector $e_{0,i}^1 \in \mathbb{R}_{++}^l$ and a twice continuously differentiable, strictly differentiably quasi-concave, and strictly differentiably monotonically increasing utility function $u_{0,i}: \mathbb{R}_{++}^l \to \mathbb{R}$. We assume that for each agent, the indifference curve containing his initial endowment bundle does not intersect the boundaries of his consumption set. The above collection of agents $\{(u_{t,i}, e_{t,i}): i \in I_t, t = 0, 1, \ldots\}$ is an economy $E$. If we fix the collection of utility functions $\{u_{t,i}: i \in I_t, t = 0, \ldots\}$, we can parametrize $E$ by the endowments $(E_1, E_2, \ldots)$ where $E_t \in \mathscr{E}_t$ is the $l(\#I_{t-1} + \#I_t)$ dimensional vector of period $t$ endowments owned by those agents born at $t-1$ and $t$, $t = 1, \ldots$.

Definition. An equilibrium $[p, x] = [(p^1, \ldots), x_{0,i}: i \in I_0, (x_{t,i}^t, x_{t,i}^{t+1}): i \in I_t, t = 1, \ldots]$ satisfies

$$\sum_{i \in I_{t-1}} x_{t-1,i}^t + \sum_{i \in I_t} x_{t,i}^t = \sum_{i \in I_{t-1}} e_{t-1,i}^t + \sum_{i \in I_t} e_{t,i}^t, \qquad t = 1, \ldots;$$

for each $(t, i)$: $i \in I_t$, $t = 1, \ldots$, $(x_{t,i}^t, x_{t,i}^{t+1})$ maximizes $u_{t,i}(x^t, x^{t+1})$ subject to the budget constraint $p^t x^t + p^{t+1} x^{t+1} = p^t e_{t,i}^t + p^{t+1} e_{t,i}^{t+1}$; for each $(0, i)$, $i \in I_0$, $x_{0,i}^1$ maximizes $u_{0,i}(x^1)$ subject to the budget constraint $p^1 x^1 = p^1 e_{0,i}^1$.

Definition. A utility function $u_{t,i}$ is intertemporally separable if and only if it can be written in the form

$$u_{t,i}(x^t, x^{t+1}) = f_{t,i}(u_{t,i}^t(x^t), u_{t,i}^{t+1}(x^{t+1}))$$

where $f_{t,i}: \mathbb{R}^2 \to \mathbb{R}$.

Suppose that, for some $T$, all the utility functions $u_{T,i}$, $i \in I_T$, are intertemporally separable. It is then possible to imagine splitting each agent $(T, i)$ into two agents, $(u^T_{T,i}, e^T_{T,i})$ and $(u^{T+1}_{T,i}, e^{T+1}_{T,i})$. This would split the economy $E$ into two disjoint economies, a finite, Arrow–Debreu economy $\bar{E}(1, T)$ with $Tl$ commodities, and an OG economy $\bar{E}(T+1, \infty)$ beginning at $T+1$. Agent $(T, i)$ in economy $\bar{E}(1, T)$ maximizes $u^T_{T,i}(x^T)$ subject to the constraint $p^T x^T = p^T e^T_{T,i}$; similarly, agent $(T, i)$ in economy $\bar{E}(T+1, \infty)$ maximizes $u^{T+1}_{T,i}(x^{T+1})$ subject to the budget constraint $p^{T+1}x^{T+1} = p^{T+1} e^{T+1}_{T,i}$.[3]

Finally observe that, if $S = \{T_1 < T_2 < \cdots\}$ is an infinite collection of generations such that $u_{t,i}$ is intertemporally separable for all $i \in I_t$ and $t \in S$, then we can define a corresponding sequence of finite, Arrow–Debreu economies $\bar{E} = \bar{E}(1, T_1), \bar{E}(T_1 + 1, T_2) \cdots$ constructed in the above manner.

PROPOSITION 1. *Let $S = \{T_1 < T_2 < \cdots\}$ be an infinite collection of generations each consisting of only one intertemporally separable agent. Then $x = (x_{0,i}\colon i \in I_0, (x^t_{t,i}, x^{t+1}_{t,i})\colon i \in I_t, t = 1,\ldots)$ is an equilibrium allocation for the* ISOG *economy $E$ if and only if $(x_{0,i}\colon i \in I_0, (x^t_{t,i}, x^{t+1}_{t,i})\colon i \in I_t, 1 \leqslant t \leqslant T_1 - 1, x^{T_1}_{T_1,i}, i \in I_{T_1}), (x^{T_1+1}_{T_1}\colon i \in I_{T_1}, (x^t_{t,i}, x^{t+1}_{t,i})\colon i \in I_t, T_1 + 1 \leqslant t \leqslant T_2 - 1, x^{T_2}_{T_2,i}, i \in T_2) \cdots$ is a sequence of equilibrium allocations for the finite, Arrow–Debreu economies $\bar{E} = \bar{E}(1, T_1), \bar{E}(T_1 + 1, T_2),\ldots$.*

*Remark.* In particular, if every generation consists of only one intertemporally separable agent, then the overlapping-generations economy is isomorphic to a sequence of one-period, $l$-commodity, Arrow–Debreu economies.

*Proof.* Let us begin with an equilibrium $[p, x]$ for the ISOG economy $E$. Note that the 0th generation spends its entire income on period 1 commodities. By simple accounting it follows that, in equilibrium, the 1st generation spends, in the aggregate, exactly the value (measured by $p^1$) of its first period endowment in period 1; that is, since $(\sum_{i \in I_0} x^1_{0,i} + \sum_{i \in I_1} x^1_{1,i}) = (\sum_{i \in I_0} e^1_{0,i} + \sum_{i \in I_1} e^1_{1,i})$ and $p^1(\sum_{i \in I_0} x^1_{0,i}) = p^1(\sum_{i \in I_0} e^1_{0,i})$, it follows that $p^1(\sum_{i \in I_1} x^1_{1,i}) = p^1(\sum_{i \in I_1} e^1_{1,i})$. Arguing inductively we obtain that, for all $t = 1,\ldots$, $p^t(\sum_{i \in I_t} x^t_{t,i}) = p^t(\sum_{i \in I_t} e^t_{t,i})$. Now, for $t \in S$, there is only one member of generation $t$ and he must, therefore, satisfy $p^t x^t_t = p^t e^t_t$—for a generation $t$ with only one agent $I_t$ is a singleton and the index $i$ can be dropped. Thus, at prices $p$, the demand of agent $t \in S$ would not be altered if the two sepate budget constraints $p^t x^t = p^t e^t_t$ and $p^{t+1} x^{t+1} = p^{t+1} e^{t+1}_t$ replaced the single budget constraint $p^t x^t + p^{t+1} x^{t+1} = p^t e^t_t + p^{t+1} e^{t+1}_t$.

[3] It would be pedantic to note that an agent $(t, i)$ in the economy $E(1, T)$, $0 < t < T$ has a utility function which is strictly monotonically increasing in only $2l$ of the $Tl$ commodities, while an agent $(0, i)$ or $(T, i)$ in only $l$.

Recalling that the utility function is intertemporally separable, we see that market clearing would be preserved if the agent $(u_t, e_t)$ were replaced by two agents $(u_t^t, e_t^t)$ and $(u_t^{t+1}, e_t^{t+1})$. The prices $\bar{p}(1) = (p^1,..., p^{T_1})$, $\bar{p}(T_1+1) = (p^{T_1+1},..., p^{T_2})$,... are thus a sequence of equilibrium prices for the Arrow–Debreu economies $\bar{E}(1, T_1)$, $\bar{E}(T_1+1, T_2)$,..., supporting the same allocation $x$.

Conversely, suppose we have a sequence of Arrow–Debreu equilibria for the sequence of economies $\bar{E}$, given by prices $(\bar{p}^1,..., \bar{p}^{T_1})$, $(\bar{p}^{T_1+1},..., \bar{p}^{T_2})$, $(\bar{p}^{T_2+1},..., \bar{p}^{T_3})$,..., and an allocation $\bar{x}$. We can consider the price sequence $(\lambda^1\bar{p}^1, \lambda^2\bar{p}^2,...)$ as a possible equilibrium price sequence for $E$, for appropriately defined positive scalars $\lambda^t$. It follows from the Kuhn–Tucker theorem that, if the scalar $\mu_{t,i}$ is chosen appropriately, an agent $(t, i)$ will demand the same bundle when faced by the single budget constraint $\bar{p}^t x^t + \mu_{t,i}\bar{p}^{t+1}x^{t+1} = \bar{p}^t e_{t,i}^t + \mu_{t,i} e_{t,i}^{t+1}$ as when faced with two budget constraints $\bar{p}^t x^t = \bar{p}^t e_{t,i}^t$ and $\bar{p}^{t+1}x^{t+1} = \bar{p}^{t+1} e_{t,i}^{t+1}$. But for $T \in S$, generation $T$ consists of only one agent, and so we can unambiguously write $\mu_T$. The scalar $\lambda^t$ can now be recursively defined as follows:

$$\begin{aligned} \lambda^1 &= 1; \\ \lambda^{t+1} &= \lambda^t, \qquad \text{for} \quad t \notin S; \\ &= \mu_t \lambda^t, \qquad \text{for} \quad t \in S. \end{aligned}$$

It is evident that, for $p = (\lambda^1\bar{p}^1, \lambda^2\bar{p}^2,...)$, $[p, \bar{x}]$ is an equilibrium for the ISOG economy $E$.

Since the theory of finite Arrow–Debreu economies is well understood, and since, as we have just shown, an ISOG economy has precisely the same equilibria as a sequence of finite, Arrow–Debreu economies, we can characterize the former in terms of the latter. We shall first use the characterization in proposition 1 to define what we mean by a regular and by a generic ISOG economy.

DEFINITION. An ISOG economy $E$ is regular if and only if it has a decomposition into finite, Arrow–Debreu economies $\bar{E} = \bar{E}(1, T_1)$, $\bar{E}(E_1 + 1, T_2)$,..., each of which is regular.[4]

[4] Geanakoplos and Brown [5] gave a definition of *-regular which applies to any OG economy and which therefore does not rely on the decomposition of Proposition 1: An economy $E$ is *-regular if and only if for any $T$ and any price sequence $\bar{p}(T+1) = (p^1,..., p^T, p^{T+1})$ which clears all market up to and including period $T$ the matrix $D_{p(T+1)}z(T)$ is of rank $Tl$, where $z(T)$ is the aggregate excess demand in the first $Tl$ commodities as a function of $p(T+1)$ and the jacobian is evaluated at $\bar{p}(T+1)$. If the economy is separable, *-regularity implies regularity in the sense of this paper.

Recall that a finite, Arrow–Debreu economy is regular if and only if the jacobian of the continuously differentiable aggregate excess demand function has rank $(l-1)$—one less than the number of commodities—when evaluated at any equilibrium price. Recall, furthermore, the theorem of Debreu [4] according to which a regular, finite, Arrow–Debreu economy has a finite number of equilibria, while, for fixed utility functions, the set of endowments for which the economy is regular is open, dense, and of measure one in the space of all possible endowments under the usual topology and normalized Lebesgue measure.

Suppose now that we have not one but a countable sequence indexed by $n = 1, 2,\ldots$ of unrelated finite, Arrow–Debreu economies, each represented by parameters $E^n \in \mathscr{E}^n$. What topology should one put on the sequence of parameter spaces $\mathscr{E} = (\mathscr{E}^1, \mathscr{E}^2,\ldots)$? Given a natural topology $\tau^n$ on $\mathscr{E}^n$, one obvious candidate is the product topology $\bar{\tau}$, in which open sets are defined as arbitrary unions of open sets of the form $\prod_{n=1}^{N} \mathscr{O}^n \times \prod_{n=N+1}^{\infty} \mathscr{E}^n$, where $\mathscr{O}^n$ is open in $\mathscr{E}^n$, $n = 1,\ldots, N$, for some finite $N$. The difficulty with this topology may be seen at once: Suppose that, for each $n$, $A^n$ is open and dense in $\mathscr{E}^n$ with respect to $\tau^n$, but that there is also, for each $n$, a nonempty $B^n \subset \mathscr{E}^n/A^n$;[5] then the set $A = \prod_{n=1}^{\infty} A^n$ is not only not open in $\mathscr{E}$ with respect to the product topology $\bar{\tau}$, but, worse still, its complement $\mathscr{E}/A$ is dense, since the product topology ignores tails. Alternatively, if each $\mathscr{E}^n$ were a metric space with metric $d^n$, one might consider using the sup-norm topology $\tau_\infty$, in which open sets are defined as arbitrary unions of open balls $B(\hat{E}, r)$, where $E = (E^1, E^2,\ldots) \in B(\hat{E}, r)$ if and only if $\sup_{n=1,2,\ldots} d^n(\hat{E}^n, E^n) < r$. However, even this $\tau_\infty$ topology has too few open sets; for let $\hat{E} = (\hat{E}^1, \hat{E}^2,\ldots) \in A$ satisfy $d^n(\hat{E}^n, \bar{E}^n) \to^n 0$, where $\bar{E}^n \in B^n \subset \mathscr{E}^n/A^n$—such a sequence exists since $A^n$ is dense in $\mathscr{E}^n$; then there is no ball $B(\hat{E}, r)$, with $r > 0$, which does not contain an element of $\mathscr{E}/A$; hence $A$ is not open in $\tau_\infty$.

We are thus led to consider an even finer topology, often used in global analysis, and variously called the box, or fine, or Whitney topology (for continuous functions).

DEFINITION. The fine topology $\tau$ for $\mathscr{E} = \prod_{n=1}^{\infty} \mathscr{E}^n$ is the topology in which open sets are arbitrary unions of sets of the form $\mathscr{O} = \prod_{n=1}^{\infty} \mathscr{O}^n$, where $\mathscr{O}^n$ is open in $\mathscr{E}^n$ with the topology $\tau^n$, for all $n = 1, 2,\ldots$.[6]

[5] Think of $A^n$ as the set of regular economies of $E^n$, and think of $B^n$ as the set of degenerate economies in $E^n$.

[6] It is clear that this fine topology is finer than both the product and the sup-norm topology.

Suppose, furthermore, that we have a natural probability measure $\mu^n$ on $\mathscr{E}^n$ (which may or may not be generated by $\tau^n$); then we can define the product measure $\mu = \prod_{n=1}^{\infty} \mu^n$ on $\mathscr{E}$ in the usual way.

DEFINITION. A set $B$ is generic if and only if it is open and dense with respect to the fine topology $\tau$, and contains a set $B'$ which is $\mu$-measurable and of $\mu$-measure 1.

We are now ready to return to the ISOG economy with fixed utility functions. Recall that $S = \{T_1 < T_2 < \cdots\}$ is an infinite collection of generations each consisting of exactly one intertemporally separable agent. Let $\mathscr{E}^t = C_l^m(a, b) = \{e \in \mathbb{R}_{++}^{ml}:\ 0 \ll a \ll e \ll b\}$ be the set of possible endowments of all agents in period $t$, where $m = \#I_{t-1} + \#I_t$. Let us give $\mathscr{E}^t$ the usual, normalized Lebesgue measure and topology.

PROPOSITION 2. *Given a family of* ISOG *economies, with fixed utility functions, parametrized by endowments* $E = (E^1, E^2,...) \in \mathscr{E} = \mathscr{E}^1 \times \mathscr{E}^2 \times \cdots$, *the set B of regular economies is generic in* $\mathscr{E}$ *with respect to the product measure* $\mu$ *and the fine topology* $\tau$.

*Proof.* It follows immediately from Proposition 1, Debreu's theorem for regular finite economies, and the definitions of the fine topology and the product measure.

PROPOSITION 3. *A regular* ISOG *economy has a sequentially finite number of equilibria; that is, for any T, the number of vectors* $\bar{p}(T) = (\bar{p}^1,..., \bar{p}^T)$ *which can be extended to equilibria* $\bar{p} = (\bar{p}^1,..., \bar{p}^T, \bar{p}^{T+1},...)$ *is finite. Thus, any small perturbation of an equilibrium price will destroy the equilibrium, no matter how the other prices are changed to compensate.*

*Proof.* Given $T$, there is $T' \geqslant T$ with $T' \in S$; if $(\bar{p}^1,..., \bar{p}^T)$ can be extended to an equilibrium it can be extended to $p(T') = (\bar{p}^1,..., \bar{p}^T, p^{T+1},..., p^{T'})$, an equilibrium for $\bar{E}(1, T_1),..., \bar{E}(T_k + 1, T')$. But, from Debreu's therem on regular, finite economies, there are only finitely many of these.

PROPOSITION 4. *For some intertemporally separable utility functions* $\{u_{t,i}: i \in I_t,\ t = 1, 2,...\}$ *and* $l \geqslant 2$, *there is an open set C of endowments in* $\mathscr{E}$, *with* $\mu(C) = 1$, *such that for every* $E \in C$ *the economy E has an uncountable number of equilibria, including an uncountable number of Pareto optimal ones, even if the economy E is regular for all* $E \in C$.

*Proof.* We know that, in a finite, Arrow–Debreu economy with two agents and at least two goods ($l \geqslant 2$), we can find utility functions $u^1$ and $u^2$

for the two aganets such that for an open set $A$ of endowments, the resulting economy has 3 equilibria; in fact this can be done even if $u_t^1 = u^2$ . Let there be in each generation one agent with utility function $u_t(x_t^t, x_t^{t+1}) = u^1(x_t^t) + u^2(x_t^{t+1})$. It follows then from Proposition 1 that, with such a utility function $u$, any sequence of one-period equilibria will support an equilibrium for the OG economy. But there are $3^{X^0}$, or an uncountable number of, such sequences. Moreover, the reader can verify easily that all these are Pareto optimal, since the price sequence satisfies the Cass–Balasko–Shell condition for optimality.[7] The probability that, in a countable number of independent draws, an event $A$ of non-zero probability occurs only finitely often is clearly zero.

It remains finally to explain why the Cobb–Douglas example computed in Balasko and Shell [3] has a unique equilibrium.

DEFINITION. Let $u: \mathbb{R}^k_{++} \to \mathbb{R}$ be a utility function and $e \in \mathbb{R}^k_{++}$ an endowment vector which yield a continuously differentiable excess demand function $z(p): \mathbb{R}^k_{++} \to \mathbb{R}^k$. The agent $(u, e)$ is a gross-substitutes agent if and only if $\partial z^j(p)/\partial p^{j'} > 0$ for all $j' \neq j$. If $k = 2l$, $u(x^1, x^2) = f(u^1(x^1), u^2(x^2))$, and $e = (e^1, e^2)$, the agent $(u, e)$ is an intertemporally separable, gross-substitutes agent if and only if both $(u^1, e^1)$ and $(u^2, e^2)$ are gross-substitutes agents.

PROPOSITION 5. *Let $S$ be an infinite ollection of generations each consisting of only one intertemporally separable agent. If every agent $(t, i)$, $t \notin S$, is a gross-substitutes agent and every agent $t$, $t \in S$, is an intertemporally separable, gross-substitutes agent, then the equilibrium in unique.*

*Proof.* It follows immediately from the decomposition given in Proposition 1 and the well-known uniqueness of equilibrium for finite, Arrow–Debreu economies with gross-substitutes agents.

## REFERENCES

1. Y. BALASKO AND K. SHELL, The overlapping-generations model. I. The case of pure exchange without money, *J. Econom. Theory* **23** (1980), 281–306.
2. Y. BALASKO AND K. SHELL, The overlapping-generations model. III. The case of log-linear utility functions, *J. Econom. Theory* **24** (1981), 143–152.
3. D. CASS, On capital overaccumulation in the aggregative, neoclassical model of economic growth, *J. Econom. Theory* **4** (1972), 200–223.

[7] Balasko and Shell [1] and Cass [3].

4. G. Debreu, Economies with a finite set of equilibria. *Econometrica* **38** (1970), 387–392.
5. J. Geanakoplos and D. Brown, Understanding overlapping-generations economies as lack of market clearing, Mimeo, 1982, presented at the NBER conference on general equilibrium, Northwestern University, March 1982.
6. T. J. Kehoe and D. K. Levine, Intertemporal separability in overlapping-generations models, *J. Econom. Theory* **34** (1984), 216–226.