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THE REVELATION OF INFORMATION IN STRATEGIC MARKET GAMES

A Critique of Rational Expectations Equilibrium

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We criticize the R E E approach to asymmetric information general equilibrium because it does not explain how information gets 'into' the prices. This leads to well-known paradoxes. We suggest a multiperiod game instead, where the flow of information into and out of prices is explicitly modeled In our game Nash equilibria (N E) (1) generalize Walrasian equilibria to asymmetric information, (2) exist generically, (3) eliminate pure speculation, (4) allow prices to reveal information and markets to become more efficient over time, (5) are consistent with the weak efficient markets hypothesis that tracking past prices is not profitable, (6) yet always lead to higher utility for better informed agents (such as experts). Throughout the paper we use one concrete game In the last section we prove that there is a broad range of games that would have the same properties

1. Introduction

Consider an economy in which agents have different levels of information concerning exogenous random states of nature. In planning his actions, what account does an agent take of what others might know about the relative profitability of his opportunities? Is there a tendency for the economy eventually to behave as if all the collective information were held in every agent's hands? And what is the role played in this by the price system? At least since Hayek this has been a central problem in economics. 'My main contention', Hayek wrote in 1937,

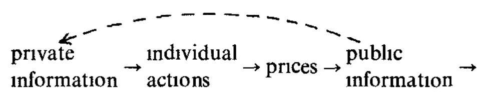
> '...will be that the tautologies, of which formal equilibrium analysis in economics essentially consists, can be turned into propositions which tell us anything about causation in the real world only insofar as we are able to fill those formal propositions with definite statements about how knowledge is acquired and communicated ... The really central problem of economics as a social science, which we pretend to solve is how the spontaneous interaction of a number of people, each possessing only bits of knowledge, brings about a state of affairs in which prices correspond

to costs, etc., and which could be brought about by deliberate direction only by someone who possessed the combined knowledge of all those individuals. Experience shows us that something of this sort does happen, since the empirical observation that prices do tend to correspond to costs was the beginning of our science. The only trouble is that we are still pretty much in the dark about (a) the conditions under which this tendency is supposed to exist, and (b) the nature of the *process* by which individual knowledge is changed... (authors' emphasis).

The "man on the spot" cannot decide solely on the basis of his limited but intimate knowledge of the facts of his immediate surroundings. There still remains the problem of communicating to him such further information as he needs to fit his decisions into the whole pattern of changes of the large economic system ... We must look at the price system as such a mechanism for communicating information if we want to understand its real function.'

One approach to this problem has been taken by the 'Rational Expectations Equilibrium' (R.E.E.) literature. It makes precise a way in which the market might communicate information through the price system, and it has provided an important explanation of certain macroeconomic and financial phenomena. But the difficulty with the R.E.E. model is that it does not raise the concomitant, and critical, question: how is information put into prices in the first place? This step in the *market process* is, as Hayek emphasized, the central problem that we must attempt to solve.

If one asks how it comes about that the diverse bits of information – held privately in the minds of individual agents – show up in prices, then the sequence of events given by the solid arrows suggests itself:



Thus the formation of prices, out of the choice of actions by all the agents in the economy, must play a key role in any discussion of the market process. Its very description invites us to model the process as a multiperiod strategic game. We choose a particular game, and analyze it for its Nash Equilibria.

There is a significant difference between the N.E. and R.E.E. approaches. Explicit in the definition of the R.E.E. model is a circularity of reasoning regarding cause and effect and their timing. The agents are supposed to know the public information revealed by the prices *before* they act to form those prices! (See dotted arrow, completing the circle.) This gives rise to well-known paradoxes in the R.E.E. model. For instance, it is possible pro forma that prices reveal more information than is held in the aggregate by all the agents – in a R.E.E. we may find that agents act with knowledge that no one has! Another feature of a R.E.E. is that private information often has no value since it is made public by prices anyway before the agents have to act.

Although these R.E.E. paradoxes can be mitigated (for example by moving to a more complicated economic model with 'noise' in the system, etc.), in contrast the N.E. approach steers clear of such paradoxes even in the simplest economic environments. Since price formation is modeled as a process, the pooling and transmission of information takes time – as any process must.[1] So when public information is revealed it can only be used in the *next* time period. The dotted arrow has no place in the game model.

The N.E. approach we take in the specific continuous game we analyze in sections 1–7 nonetheless retains many of the desirable features of R.E.E. First, it reduces to Walrasian analysis when there is no asymmetric information between agents. When information is disparate, an N.E. generically exists. Second, in an N.E. a player takes into account what others might know, and what he reveals to them, during the course of the play of the game. He also knows that they know that he knows that..., i.e., it is common knowledge that every player is rational. This implies that there can be no gains from pure speculation in an N.E., a fact which also holds for R.E.E. (and, indeed, is viewed as a major advantage of the R.E.E. analysis over the Walrasian).

Third, prices reveal information in an N.E. to all the agents and thus tend to bring about efficient markets. But since the transmission of information (both into and out of prices) takes time in the game, agents who have superior information can earn initial profits from it. Thus the 'strong efficient markets hypothesis' (which says that expert knowledge is useless!) never holds for N.E. This is in contrast to the R.E.E. situation where (in the simplest economic environment–finite state space) the strong hypothesis is often fulfilled. On the other hand, the 'weak hypothesis' (i.e., that charting past prices cannot yield information not already provided by current prices) holds for both N.E. and R.E.E. (See section 6.)

In sections 1–7 of the paper we mostly work, for concreteness, with a specific game mechanism due to Shapley and Shubik. But no one mechanism can possibly describe the variety of market processes that occur in real life. It might well be asked to what extent our results hinge on any particular choice of the mechanism. We show that in fact our analysis remains intact across a wide group of 'smooth' mechanisms (section 6). Even replacing smoothness with a weak continuity requirement on the mechanism, the N.E. approach retains its distinction from the R.E.E. (section 8). Precisely: given any continuous mechanism, there is an open set of economies, each of which has R.E.E. that cannot be achieved via the mechanism in question.

Many readers will think that the well-known device of permitting agents'

[1] No matter how short, there is a decision–information sequence, as in our figure.

strategies to include entire demand functions allows for the implementability of R.E.E. as N.E., thus blurring the distinction between the two approaches. It is to these readers especially that section 8 is addressed. We regard continuity, especially in the idealized model of a continuum of agents, as a requisite property of any sensible mechanism. Otherwise deviations by an arbitrarily small (but positive) measure of agents could have a large effect on the terms at which all the agents trade. Section 8 shows that no continuous mechanism, including some still more complicated than the demand function variety, can implement the R.E.E. correspondence. This remains true if we restrict attention to the open set of regular economies with a unique R.E.E. (that varies continuously with the underlying economy). The reason of course is that the demand function game does not specify a unique outcome when there are several market clearing prices. Even when the R.E.E. that need to be implemented are uniquely defined, the demand functions that would be required to implement them inevitably involve multiple market clearing prices, and thus a discontinuous mechanism.

Perhaps a more basic, if less theoretical, reason why we reject the arbitrary demand function game is that the strategies are so complex, and the role of the auctioneer-mechanism so complicated, that we regard it as unplayable. The Shapley–Shubik mechanism we study here also involves the submission of demand functions, but they are taken to be hyperbolic, and hence to depend on only one parameter. We could just as easily have assumed that demands are linear, that is specifiable by two parameters, a (negative) slope and an intercept, without changing any of our conclusions. (In both mechanisms demand depends only on own price and is downward sloping, hence the mechanisms are continuous.) There are many low-dimensional, continuous mechanisms whose N.E. are competitive equilibria of economies with symmetric information. None of them, however, generally yield R.E.E. of economies with asymmetric information. Sections 6 and 8 make clear that either low-dimensionality, or continuity is alone enough to maintain the contrast between the N.E. and R.E.E. approaches.

2. Definitions and properties of R.E.E.

Since our paper is juxtaposed to the R.E.E. model, and meant to provide an alternative as well as to be a critique of it, let us first briefly recall what the R.E.E. model is.[2] Let S be the finite set of states of the world, and for each agent $n \in N$ let I^n be a partition of S representing the information of n. If $s \in S$ is the actual state of nature, then each agent $n \in N$ is informed of that set $I^n(s)$ in his partition that contains s. The collective information of all the

[2]We outline here a somewhat more general version of the Radner (1979) model The main early contributors to the development and application of the notion of R E.E are Muth, Lucas (who introduced rational expectations equilibrium), Green and Grossman

agents is denoted by $I^* = V_{n\in N} I^n$, the coarsest partition of S which refines each I^n, $n \in N$.

Trade takes place in a finite set L of commodities: Thus the space of state-contingent commodities is $\mathbb{R}_+^{L\times S}$. An allocation x^n for trader n can be thought of as a function $x^n: S \to \mathbb{R}_+^L$ associating to each state s a bundle $x_s^n \in \mathbb{R}_+^L$. Each agent n is characterized by a utility $u^n: \mathbb{R}_+^{L\times S} \to R$ and an endowment $e^n \in \mathbb{R}_+^{L\times S}$. Assume u^n is C^2, strictly concave and monotonic, and that e^n is measurable with respect to I^n. Since $e^n \in \mathbb{R}_+^{L\times S}$, we can also think of e^n as a function from S to $\mathbb{R}_+^L$, associating to each state $s \in S$ an endowment $e_s^n \in \mathbb{R}_+^L$. By measurable we mean that if agent n cannot distinguish states s and s', in other words if $I^n(s) = I^n(s')$, then $e_s^n = e_{s'}^n$. Hence knowing the function e^n, agent n also always knows his endowment, for given any $s \in S$ he is informed of $I^n(s)$ and since $e_{s'}^n$ is the same for all $s' \in I^n(s)$, he can deduce what the realization e_s^n must be.

Consider a price function $p: S \to \mathbb{R}_+^L$. Its inverse yields a partition of S which we will denote by $I(p)$. Thus for any realization $\bar{p}$ of p, $I(p)(\bar{p}) = \{s \in S: p_s = \bar{p}\}$. Notice that the information agent n can deduce from prices depends on knowing the price function p and the particular realization $\bar{p} = p_s$. Such an agent n would have access to knowledge denoted by the partition $I^n \vee I(p)$. In R.E.E. it is assumed that every agent n knows his utility u^n, his endowment function e^n, the price function p, and the price realization p_s before he must act in any state s. A R.E.E. for this economy is a p, along with allocations $x^n: S \in \mathbb{R}_+^L$ such that, for $n \in N$,

(i) x^n is measurable w.r.t. the coarsest refinement $I^n \vee I(p)$ of I^n and $I(p)$,
(ii) $x^n = \text{argmax}_{x \in \mathbb{R}_+^{L\times S}} \{u^n(x): x$ is measurable w.r.t. $I^n \vee I(p)$, and $p_s \cdot x_s \leqq p_s \cdot e_s^n$ for each $s \in S\}$,
(iii) $\sum_{n\in N} x^n = \sum_{n \in N} e^n$.

(Here, for any vector $x \in \mathbb{R}_+^{L\times S}$ and $s \in S$, x_s is the vector in $\mathbb{R}_+^L$ obtained by restricting x.) In words this means that each agent n refines his information I^n by what he can deduce from seeing p_s, then forms his demand x_s^n (subject to the budget constraint $p_s \cdot x_s^n \leqq p_s \cdot e_s^n$), and the ensuing total demand $\sum x_s^n$ and can be met by the supply $\sum e_s^n$ at hand. Notice that prices play the dual role of simultaneously determining the budget constraint and revealing information. Note also that in this definition of R.E.E. the utilities u^n are unaffected by the information: traditional definitions of R.E.E. have agents maximizing with respect to conditional utilities, i.e., utilities modified by the information. In our definition the role of information is captured by the measurability restriction. Our definition is more general, since it does not presume Von Neumann–Morgenstern utilities, but it incorporates the other as a special case: an agent who can distinguish s from s' but is constrained to act the same way in both states will choose exactly as he would if he did not have that information.

Let us consider an example illustrating the difference between the standard definition of Walrasian equilibrium with uncertainty, in which prices serve only to fix exchange rates and do not convey information, and a R.E.E.

Example 1. Suppose there are two states of nature, two goods, and two agents, both with the same utility functions:

$$U(x_{s_1}, m_{s_1}, x_{s_2}, m_{s_2}) = \tfrac{1}{2}(m_{s_1} + 0x_{s_1}) + \tfrac{1}{2}(m_{s_2} + 2x_{s_2}).$$

Let agent 1 own 1 unit of the x good in both states and let agent 2 own $\bar{M} < 1$ units of the m good in both states. Notice that the sale of the x good is in effect a bet between the agents over whether the state s_1 will occur, or not. Let agent 1 know which state has occurred, and let agent 2 be uninformed. Let the price ratio of x to m be $p_{s_1} = \bar{M}$ and $p_{s_2} = 2$. If agent 2 ignores the informational content of prices, then this is a competitive equilibrium: in state 1 agent 2 thinks he is making a favorable bet and will sell off all his m in order to purchase 1 unit of x; agent 1, on the other hand, knows that x is worthless and gladly sells his unit holding of x. In state 2 agent 2 demands no x and agent 1 will demand precisely his initial endowment. Clearly agent 2, under this definition of equilibrium, ends up betting only when he is sure to lose.

The agents are *not* in R.E.E., however, for in state 1, seeing the realization $\bar{p} = \bar{M}$, agent 2 would infer that the state was indeed s_1 and would *not* be willing to give up m in exchange for x. The only R.E.E. is given by the price function $p_{s_1} = 0$ and $p_{s_2} = 2$, and allocations $x^n = e^n$ that involve no trade.

As this example suggests, one of the desirable properties of a R.E.E., vis-à-vis the naive definition of Walrasian equilibrium with uncertainty, is that in a R.E.E. no agent can end up with an allocation that is less good (ex ante) than his endowment e^n. The reader can immediately verify this individual rationality property from our definition of R.E.E. – the endowment e^n is by hypothesis measurable with respect to I^n and it clearly satisfies the budget constraint. The no speculation result proved so often in the literature is also transparent from our definition of R.E.E.[3] Let an allocation $\{y^n\}$ be called ex-ante Pareto optimal if it is Pareto optimal in the above economy with respect to u^n, i.e., before any information is obtained. Suppose $\{e^n\}$ is ex-ante Pareto optimal, so that there are only speculative gains to trade, and let $\{x^n\}$ be a R.E.E. allocation. Then $x^n = e^n$ for all $n \in N$. This follows at once from the fact that each agent n could have chosen e^n, hence $u^n(x^n) \geqq u^n(e^n)$ for all n, hence $x^n = e^n$ for all n.

[3]The more traditional definitions of R.E.E. (with their emphasis on conditional probabilities) were made in a narrower context than ours. An advantage of our more general definition is that it makes the no speculation result both more widely true and much easier to prove

Example 1 also illustrates the proposition of Radner that when the state space S is finite, then generically (in utilities and endowments) R.E.E. exist, are finite in number, and are fully revealing, in the sense that every agent can infer from prices alone the combined knowledge of all the traders, i.e., that $I(p) \supset I^*$.[4] Of course in the special case where there is no uncertainty, or information is symmetric, R.E.E. and the usual Walrasian equilibria coincide. Thus R.E.E. generalizes Walrasian equilibrium to economies with differing levels of information, it permits traders to prudently take into account what others may know about them, and it explains how the market can communicate to each agent the combined knowledge of all.

3. Difficulties with R.E.E. and its extensions

Nevertheless we ask how can it be that agents take into account the prevailing prices $\bar{p} = p_s$ in choosing their actions when it is precisely those actions which create the prices?[5] Without an explicit explanation of how prices are formed, we argue, there can be no theory about what prices can and cannot reveal. Indeed in the above definition when $S \supsetneqq I^*$, there will typically be R.E.E. with the absurd property that $S = I(p) \supsetneqq I^*$, i.e., that prices reveal information that no agent possesses!

Although this paradox disappears if we arbitrarily (without a model of price formation) restrict our attention to R.E.E. for which $I(p) \subset I^*$, we are still confronted with another paradox. In R.E.E. agents have no incentive to gather costly information that they will be able to infer for free from prices. If S is finite then, by Radner's theorem, R.E.E. prices will fully reveal all private information. [$I(p) = I^*$.] But then, *each* individual trader in a continuum would rather learn from these prices than privately collect information. The upshot is that no information would be gathered for prices to reveal. [See the example in Grossman and Stiglitz (1980).] To avoid this paradox, several modifications of the Radner model we outlined above have been proposed. In noisy R.E.E. models, introduced in Lucas (1972) and Grossman (1977) and treated more generally by Anderson and Sonnenschein (1982) and Allen (1982), the state space S is taken to be infinite, so large in fact, that prices cannot be fully revealing. Although this mitigates the information gathering disincentives of a fully revealing R.E.E., it remains true

[4]Beth Allen (1981) has generalized this result to the case where the state space may be infinite, if it has dimension less than half that of the price space. Radner's formulation of a generic event is slightly different from the above, but his proof can easily be used to obtain the result we quote.

[5]It will not do, especially in an environment where information plays a crucial role, to refer to a tatonnement procedure through which agents grope to $\bar{p} = p_s$, and then make inferences about s, for the procedure itself will communicate information In any case, tatonnement, with its false trades and so on, is an imaginary process. Nevertheless, it has been studied with care and precision by Jordan (1982b).

that the more efficiently prices reveal information, the less is the incentive to gather information in R.E.E. Adding noise to the system creates almost as many problems as it solves. To prove the existence of R.E.E., one often drops the requirement that demand equals supply. That can be replaced either by an ε-market clearing [as in Allen (1982)] or by allowing unmodeled, random perturbations to demand and supply [as in Grossman (1977) and Anderson and Sonnenschein (1982)]. In the latter case, one must also weaken the rationality hypothesis to approximate rationality. Among other things, this implies that the individual rationality principle and the no speculation theorem can be violated. The Anderson–Sonnenschein (1982) paper recognizes the validity of the N.E. approach, implicitly deriving conditions under which the arbitrary demand function game has approximate N.E. This represents already a partial abandonment of the R.E.E. approach, since these approximate N.E. do not involve strict market clearing or strict rationality. We also observe in section 3 that the mechanism is not continuous. Let us also note that the mechanism is enormously complicated: demand for each commodity is conditional not only on its price, but on the prices of all other commodities as well. By contrast we offer a simple (i.e., low dimensional) continuous mechanism which maintains both strict market clearing and strict rationality.[6]

4. The N.E. approach

In this paper we consider a model with an explicit process for the flow of information via prices. Roughly it goes as follows. Economic activity takes place in time periods. Agents initially act on the basis of their privately-held information I^n. This results in observable economic outcomes (e.g., prices) through which their information is 'betrayed'. The extra information so released to everyone is then available for the *next* period of activity. Notice that in the initial period agents with superior information can exploit it and make a 'killing'. The paradox that information is useless is removed by the simple fact that the process that reveals it takes time.

This description is, we believe, more realistic than the R.E.E. model (and also more in keeping with what Hayek had in mind). It suggests the use of a strategic market game as the appropriate model. We shall, for concreteness, choose one such: the model of exchange presented in Dubey and Shubik (1978) extended here to allow for asymmetric information and many time

[6]Hellwig (1982) also introduced an alternative model, in which traders learn from past prices, and observe current prices to calculate their budget constraints. However, they are artificially forbidden to infer information from the current prices, so the individual rationality property is violated in precisely the same way it is in Example 1. Let us mention that Milgrom (1981) has criticized the notion of R.E.E in the context of an auction of indivisible objects.

periods. But as we emphasize later, our results seem to be quite robust and not to hinge delicately on this choice (see section 8).

Before plunging into the analysis in the next section, we outline here the contours of our model, first very sketchily and then with some of the details. S, N, $\{I^n\}_{n\in N}$ are as before. But now there are time periods, for simplicity two. The characteristics of the traders must accordingly be expanded into endowments e^n, $\tilde{e}^n \in \mathbb{R}_+^{L\times S}$ in period 1(2); and utility $u^n: \mathbb{R}_+^{L\times S} \times \mathbb{R}_+^{L\times S} \to \mathbb{R}$. In essence, in the first period players must irrevocably choose to put commodities up for purchase or sale solely on the basis of their own information. These moves automatically lead in a simple way to prices and allocations. In the second period the traders must make similar choices, this time with the benefit of having observed first period prices, leading to second period prices and allocations. If there is no ex-ante asymmetric information, and traders have enough 'money' [see Dubey and Shapley (1980)] then this game produces the Walrasian equilibria as its Nash equilibria.[7] On the other hand, when players are not fully informed they face two sources of uncertainty in their first period activity: what the terms of trade (i.e., prices) will be and what the utility value of the commodities will be (since utility depends on the state). In the second period both uncertainties will be reduced by the observations of first period prices, and marginal utilities (costs) will be brought closer together. Although we shall illustrate our game with examples in section 7, it might be helpful at this point to introduce some of the details of our model.

The game is best viewed in extensive form. Nature moves first to select a state s in S. At each node s all the players in N move simultaneously with information partitions given by I^n. A move for a player $n \in N$ at node s is a $2(L-1)$ dimensional vector of bids and offers $z^n = (b_1^n, \dots, b_{L-1}^n, q_1^n, \dots, q_{L-1}^n)$. Each b_i^n represents a quantity of the Lth good that is bid on commodity i, $i = 1, \dots, L-1$, and each q_i^n represents a quantity of the ith good that is offered for sale, $i = 1, \dots, L-1$. Given a vector[8] $z = (z^n)_{n\in N}$ of moves of each player $n \in N$, the market performs the computationally trivial task of adding[9] the bids for each good, $b_i(z) = \sum_{n\in N} b_i^n$, $i = 1, \dots, L-1$ and the offers, $q_i(z) = \sum_{n\in N} q_i^n$, $i = 1, \dots, L-1$. It sets[10] $p_i = \pi_i(z) = b_i(z)/q_i(z)$. The consumers then receive the net trades $y_i^n = b_i^n/p_i^n - q_i^n$, $i = 1, \dots, L-1$, $n \in N$ and $y_L^n = \sum_{i=1}^{L-1} q_i^n p_i - \sum_{i=1}^{L-1} b_i^n$, $n \in N$, where $p_i = \pi_i(z)$, etc. If there is no inventorying of commodities between periods 1 and 2, and if z is the vector of moves

[7] Our critique that the R.E.E. omits the process of price formation applies to Walrasian equilibria. But, while the Walrasian equilibria can be recovered from a process oriented model (e.g., the Shapley–Shubik model) the R.E.E. can not.

[8] If N is a continuum, think of (i) z as a measurable function, (ii) $\sum_{n\in N} b_i^n$ as $\int_N b_i^n \, dn$, etc.

[9] See footnote 8.

[10] Take division by zero to be zero throughout.

chosen by the players at node $s \in S$, then the consumption of agent n in period 1 in state s is equal to his net trade at node s plus his initial endowment: $x_i^n(s) = e_i^n(s) + y_i^n$, $i = 1, \ldots, L$. Each node s in the tree leads to an infinity of successor nodes (s, z), where z is a vector[11] of moves, one for each player. Play proceeds from node s to node (s, z) if the players collectively choose the move z at node s. At each second period node (s, z) the players again move simultaneously, choosing moves $\tilde{z}^n$ exactly as in period 1, with information partitions $\tilde{I}^n$ given by $(s, z) \sim (s', z')$ if and only if $I^n(s) = I^n(s')$ and $\pi(z) = \pi(z')$. The terminal nodes are all of the form $(s, z, \tilde{z})$. Play proceeds from (s, z) to its successor $(s, z, \tilde{z})$ iff each player n plays the move $\tilde{z}^n$ at node (s, z). Given a terminal node $(s, z, \tilde{z})$, we calculate first period consumption of agent n in state s as above, and second period consumption in state s in exactly the same way, with $\tilde{e}^n(s)$ replacing $e^n(s)$ and $\tilde{z}$ replacing z. Notice that the market (which embodies the rules of our game) has a simple task, which can be performed without any knowledge of the state s, or the private characteristics of traders n, or even of fixed point theory.

A strategy of a player $n \in N$ is to pick a move $z^n(s)$ in period 1 at each node $s \in S$ in such a way that the function[12] $z^n : S \to \mathbb{R}_+^{2(L-1)}$ is measurable with respect to I^n, and to pick a move $\tilde{z}^n(s, z)$ at each node (s, z) in such a way that the function $\tilde{z}^n : S \times Z \to \mathbb{R}_+^{2(L-1)}$ is measurable with respect to $\tilde{I}^n$. We also require that the function z^n and $\tilde{z}^n$ satisfy the further property that $q_i^n(s) \leqq e_i^n(s)$, $\tilde{q}_i^n(s, p) \leqq \tilde{e}_i^n(s)$, $\sum_{j=1}^{L-1} b_j^n(s) \leqq e_L^n(s)$, $\sum_{j=1}^{L-1} \tilde{b}_j^n(s, z) \leqq \tilde{e}_L^n(s)$, $i = 1, \ldots, L-1$, $n \in N$.[13] Given a vector of strategies $\bar{z} = (z^n, \tilde{z}^n)$ for each player $n \in N$, a terminal node is determined for each $s \in S$, and hence first and second period allocations $x^n(z(s))$, $\tilde{x}^n(s, z(s))$, and first and second period prices $\pi(z(s))$, $\pi(\tilde{z}(s, z(s)))$ for each $s \in S$. We analyze this game for its Nash equilibria. In section 7, we present easy-to-calculate examples of our game.

5. Results and interpretations

Although we shall formally state and prove our results in section 6, they may be summed up as follows. In any Nash equilibrium (N.E.) the resulting allocations $(x^n, \tilde{x}^n)$ are at least as good for each $n \in N$ as the initial endowments $(e^n, \tilde{e}^n)$, hence the individual rationality principle is maintained.

[11] See footnote 8.

[12] Note that now z^n is a function, while in the previous paragraph it was a realization of this function at a particular node s. No confusion should result

[13] Since players are assumed to know their endowments, these restrictions are permissible One interpretation of the restrictions is that the players must physically send their bids and offers to the market Alternatively we could allow players to send paper bids and offers We would then have to specify the allocation which would result if some of the players defaulted on their offers (with asymmetric information it would not be known if a player made a promise he could not keep). We would also specify a penalty for a player who defaulted If the penalty were made sufficiently harsh, no player would ever default in equilibrium.

Furthermore, if N is non-atomic, then for a generic choice of e^n, $\tilde{e}^n$ and u^n: N.E. exist and are finite in number; they fully reveal I^* in that $\pi(z(s)) \neq \pi(z(s'))$ if $I^*(s) \neq I^*(s')$, and lead to higher utilities for the better-informed agents. If N is finite then generic revelation fails, and N.E. exist robustly (i.e., for an open set of e^n, $\tilde{e}^n$ and u^n) at which some agents do not betray all their information in the first period. Thus generic revelation by prices is a phenomenon that attaches to perfect competition and is seen to break down in the oligopolistic setting. The non-atomic case of perfect competition is simpler [as is shown in general in Dubey and Kaneko (1982)] in that the strategies can be taken to depend on history only insofar as that history reveals something about the state of nature. Threat equilibria, in which strategies depend also on the moves of other agents, disappear. To return to our main point, in the non-atomic case agents rationally take into account what others know, and the market is efficient in the sense that prices eventually convey all relevant information. Yet we have a *process* which takes real time and always allows the better-informed traders to profit from their superior information. Technical analysis (the study of past prices) does not bring superior rewards, as we shall see in section 6; contrary to the strong forms of the efficient markets hypothesis, however, both fundamental analysis and insider information are potentially profitable (see again section 6).

One can give several interpretations to the time periods in our model. For example they might correspond to the seasonal meetings of the market after harvest. The farmer offers to the market his crop of wheat, as a matter of prior commitment made at the time of planting, no matter what the price turns out to be. When he is deciding how much to plant he looks to the previous period's price as a guide to what price his wheat might bring in the future. In that sense the farmer in our model is a more sophisticated analogue of the farmer in the famous cobweb model, who always assumed next period's price would be the same as last period's price. It might be natural under such an interpretation of the periods in our model to take the utilities $u^n(x, \tilde{x})$ to be of the form $v^n(x) + w^n(x)$. It is easy to see (and Example 2 in section 7 illustrates it), that with such separable utilities the second period N.E. prices $\tilde{p}(s)$ and allocations will (generically) be exactly the same as the R.E.E. prices and allocations in the economy $E = (\tilde{e}^n, w^n)_{n \in N}$, not because agents learn from prices $\tilde{p}(s)$, but rather because they already learned from first period prices $p(s)$. Of course it might still be more natural to allow the money good L to be inventoried from the first period to the second period, and perhaps to assume that each farmer quotes a price p^n as well as a quantity and that consumers then choose where to buy and so on. As we have indicated earlier, all these extensions to our model can be easily incorporated (for example by using the Dubey–Shubik [(1978, 1980)] price–quantity models instead of the Shapley–Shubik bid–offer model) without affecting our basic results.

In a second interpretation we might take our model as a representation of a tatonnement process with actual (not 'virtual') trade. Our innovation is not in postulating a dynamic model in which recontracting is not allowed, but rather in providing a model in which agents optimize over the whole horizon, not myopically, and in which they, not some fictitious auctioneer, set the prices.[14] Of course under such an interpretation one should allow almost all the goods to be inventoried. (Again, as long as at least one good cannot be inventoried in any state, all of our results remain unaffected.) One would also expect that the utilities $u^n(x,\tilde{x})$ would be $v^n(x+\tilde{x})$, i.e., the disutility of waiting is negligible. Since our main theorem applies only generically, and perfect substitutes utilities are a knife-edge case, our propositions do not necessarily apply, when $u^n(x,\tilde{x})$ is $v^n(x+\tilde{x})$. [They do apply, however, to utilities $u^n(x,\tilde{x})$ arbitrarily close to $v^n(x+\tilde{x})$.] Therefore one might suspect that the agents would trade very little in the first period and simply wait until the second period when they had more information to do most of their trading. The resulting N.E. allocations might then look very much like the R.E.E. allocations of the economy $E=(v^n, e^n+\tilde{e}^n)_{n\in N}$ that we have been criticizing. As we shall show in the last example of section 8, where we take $u^n(x,\tilde{x})=v^n(x+\tilde{x})$ and we allow all goods to be inventoried, this intuition is wrong. Agents would trade in both time periods because the prices in general will be different. An agent may know better in period 2 what is really valuable, but so does everyone else, and the price may be higher. There is an insurance benefit to trading before the information causes the prices to fluctuate. In fact, for this reason the allocation of the strategic market game in our particular example Pareto dominates the R.E.E. allocation.

In our game nature moves only before the first period; by the second period the less well informed catch up to the better informed. It would be just as simple to consider a model in which a new event occurs every period. In that case 'experts' would remain one step ahead of everybody else.

Let us re-emphasize that our results are not at bottom an artifact of the model we have employed. We could, for example, have had agents submitting limit orders, contingent on several prices, and still proved similar results, provided that the dimension of the strategy spaces was not too large (see section 6). No matter how large the strategy spaces, if the mechanism is weakly continuous, it cannot implement R.E.E. (see section 8). Consider the excess demand function mechanism. It is easy to show that if N is non-atomic and if the strategy space includes all excess demand functions, and if the rules of the game specify that trade takes place at the unique market

[14] The most prominent myopic models, which place fewer demands on the calculating abilities of the agents, are Bray (1982) and Blume and Easely (1982). Nevertheless, these models are open to the same process criticism we have been leveling against R.E.E.

clearing price given all the demands if there is one (otherwise no trade is allowed), then generically R.E.E. can be implemented as N.E. Of course this mechanism is open to the obvious criticism that one cannot imagine agents (or a clearing house) who have the capacity needed to play it. We take as a dictum – and this is met by our model – that both the strategy sets and the outcome map be simple and 'playable'. The mechanism is also not continuous: if a few traders change their demands, the allocation can jump wildly (from zero net trade to a large trade). Observe, incidentally, that the submission of a fixed bid b_i is equivalent to the submission of an entire demand curve, $\{x(p)|x\cdot p=b_i\}$, that is everywhere downward sloping. The submission of a fixed quantity for sale q_i is of course equivalent to a vertical supply curve. Given a collection of such downward sloping demand curves and supply curves (all, by the way, depending only on the own price) a small change in one of them will inevitably lead to a small change in the equilibrium price and allocation. In order to implement R.E.E. over more than a small domain of economies, however, one would need upward sloping demands as well as downward sloping demands. And as section 8 makes clear, that inevitably leads to a discontinuity.

6. Formal statement of the N.E. approach

We consider the case when the agent-space is non-atomic. For convenience there is a finite number of types of agents: $1,\dots,N$. Type n consists of the continuum $(n-1,n]$ endowed with the Lebesgue measure for every $n\in N=\{1,\dots,N\}$. [The triple use of n: as the number n, as the set $(n-1,n]$, as the name of the nth type; as well as the additional fourth use of N as the set of types $\{1,\dots,N\}$; should cause no confusion. The usage will always be clear from the context, and it saves enormously on notation.]

The game Γ described in section 4 has some trivial 'inactive' N.E.'s; for instance consider the strategies in which all agents bid or supply zero everywhere. Additional N.E.'s can be constructed which leave any specified subset of the $2\times(L-1)\times S$ trading posts inactive. Our interest is in pinning down conditions which guarantee the existence of *active N.E.'s*, namely those which produce positive prices in each trading post. From now on, we shall always mean an active N.E. when we say N.E.

It turns out that N.E. do not always exist for Γ. However we will state sufficient, and rather weak, conditions on endowments and utilities for which N.E. exist generically. Let $\mathscr{E}$ be a cube in $\mathbb{R}_+^{N\times L\times S}\times\mathbb{R}_+^{N\times L\times S}$ which is bounded and has no vector with any zero components. Each point in $\mathscr{E}$ completely specifies endowments of the L goods in each of the S states and the two time periods across the N trader-types. If utilities are strictly concave, then any N.E. of our game will be type-symmetric. Therefore we confine our analysis to type-symmetric reallocations. Clearly there is a cube

$\mathscr{C}$ in $\mathbb{R}_+^{L\times S}\times\mathbb{R}_+^{L\times S}$ such that if $x^1,\ldots,x^N$, $\tilde{x}^1,\ldots,\tilde{x}^N$ is such a reallocation of a point in $\mathscr{E}$, then $(x^n,\tilde{x}^n)\in\mathscr{C}$. Let $\mathscr{U}$ be a finite dimensional manifold of C^2, strictly concave functions defined on some neighborhood $\mathscr{N}$ of $\mathscr{C}$ which satisfy, for some $0<\sigma<\sigma'$, $\sigma<\partial U/\partial x_{is}$, $\partial U/\partial \tilde{x}_{is}<\sigma'$.[15] Suppose also that if $u\in\mathscr{U}$ and $(c,\tilde{c})$ is any sufficiently small vector in $\mathbb{R}^{L\times S}\times\mathbb{R}^{L\times S}$ then $v(x,\tilde{x})\equiv u(x,\tilde{x})+(c,\tilde{c})\cdot(x,\tilde{x})$ is also in $\mathscr{U}$. The simplest example of such a space $\mathscr{U}$ is obtained by fixing a single C^2, monotonic, strictly concave $\bar{u}$ and letting $U=\{v:v=\bar{u}+(c,\tilde{c})\cdot(x,\tilde{x}),\ \|c,\tilde{c}\|<\varepsilon\}$. A point in $\mathscr{U}^N$ represents a choice of utilities for the N types. We will keep all the other data of the game fixed as in section 4, and vary only the endowments and utilities. $\mathscr{E}\times\mathscr{U}^N$ can then be thought of as the space of games. Our main theorem is now readily stated.

Theorem 1. There is an open dense set $\mathcal{O}$ in $\mathscr{E}\times U^N$ with the properties:

(i) *N.E. exist, are finite in number, and vary continuously on $\mathcal{O}$.*

(ii) *If $z:(0,N]\to\mathbb{R}_+^{(L-1)\times S}+\mathbb{R}_+^{(L-1)\times S}$ is the move at any N.E. in (i), then z is fully revealing, i.e.,*

$$I^*(s)\neq I^*(s')\Rightarrow \pi(z(s))\neq\pi(z(s')).$$

(iii) *Suppose there are two types of traders n and n' who are identical in all respects except that I^n strictly refines $I^{n'}$. Then at each N.E. in (i), the final utility of type n traders is strictly bigger than that of type n'.*

We defer the proof until section 9. Here let us examine the bearing of the theorem on the *efficient markets hypothesis*. This comes in both a strong and a weak form. The strong form asserts that individual expert knowledge cannot be exploited. Part (iii) of our theorem asserts just the opposite, sharply distinguishing the N.E. approach from the R.E.E.

To discuss the weak form, let us imagine that our game is played over more than two time periods: $1,2,\ldots,T$. Each agent n could be supposed to have fixed initial private information as before, or more generally, he might have access to successively finer information: $I^n(1),I^n(2),\ldots,I^n(T)$, where $I^n(t)$ refines $I^n(t-1)$. In addition, at any time t an agent can use the entire history of past prices $p(1),\ldots,p(t-1)$, so that his knowledge is given by $I^n(t)\vee I(p(1))\vee I(p(2))\vee\cdots\vee I(p(t-1))$. It can be shown in this game that, at a generic N.E.,

$$I(p(t-1))\quad\text{refines}\quad I(p(t-j))\quad\text{for } j=2,\ldots,t-1,\quad t=2,\ldots,T.$$

[15]In fact we could take $\mathscr{U}$ to be the infinite dimensional Banach manifold of all such functions, and get the same result.

(The proof is entirely analogous to the proof of Theorem 1 in section 9.) Therefore in an N.E. no agent can gain from tracking the history of prices; all that he could learn is embodied fully in the last period prices. This is exactly the weak form of the efficient markets hypothesis, and our N.E. satisfy it. (So do the R.E.E.)

Remark 1. The space of utilities U can be taken to be all C^2 functions on $\mathcal{N}$, equipped with the C^2 topology. Theorem 1 remains true; we would need to use the infinite dimensional transversality theorem in its proof.

Remark 2. Theorem 1 remains intact for a wide variety of 'smooth' games, though we focused on the Shapley–Shubik model for concreteness. Suppose that the set of moves Σ^n available to type n agents is an open, bounded set in some Euclidean space (for each n). We consider only type-symmetric choices of moves and require that the map from $\Sigma^1 \times \cdots \times \Sigma^n$ to prices and trades be smooth. Finally suppose that the dimension of $\Sigma^n < (L-1)\cdot \max\{|T| : T \in I^n\}$ for some n. Then our theorem goes through by a similar proof. (If one considers the potentially large number of states of nature, in any realistic situation, then the restriction on the dimensionality of Σ^n is seen to be not so severe.)

7. Some examples

In the first three examples there are three types of agents and two goods, a commodity good and a money. There are two time periods and two states of nature. In the second state of nature the commodity has no value to any trader. There is a continuum $(2,3]$ of identical agents $\alpha \in (2,3]$ called sellers who each own 20 units of the good in each period and no money. They have utility only for money. They will always put all their goods on sale in our simple Shapley–Shubik game and, since they have no money to bid, we can suppress their choice of actions from our analysis of the strategic game.

The first two types of traders have the same utility functions

$$u^i = \tfrac{1}{2}(A \log x_1^i + w_1^i + B \log \tilde{x}_1^i + \tilde{w}_1^i) + \tfrac{1}{2}(w_2^i + \tilde{w}_2^i)$$

$$= [\tfrac{1}{2}(A \log x_1^i + w_1^i) + \tfrac{1}{2} w_2^i] + [\tfrac{1}{2}(B \log \tilde{x}_1^i + \tilde{w}_1^i) + \tfrac{1}{2}\tilde{w}_2^i] = \Pi + \tilde{\Pi}, \qquad (1)$$

where x_1^i ($\tilde{x}_1^i$) is the consumption of the good in state 1 at time one (two) and w_s^i ($\tilde{w}_s^i$) is the holding of money during time period one (two) in state s, $s-1$ or 2. Each of these traders has an endowment vector $(0, M)$ in both periods. The trader(s) of type 1 can distinguish between states $s=1$ and $s=2$, while the trader(s) of type 2 are uninformed.

Example 2. The purpose of this example is to illustrate Theorem 1 and to show that the N.E. approach is easy to work with. We assume that there is a continuum of agents $\alpha \in (0,1]$ of the first type and also a continuum of the second type $\alpha \in (1,2]$. For now we shall not allow the inventorying of either good (including the money). In the first period the uninformed traders will each bid an amount b in both states of nature, while the informed traders will bid state dependent amounts b_1 and b_2. If we have not made a degenerate choice of utilities, the resulting prices p_1 and p_2 will, according to our theory, be different and so will reveal the state of nature to the uninformed traders in period 2.

Since there is a continuum of traders of each type and the utilities are separable between time periods, our introductory remarks imply that a player α will make his first period move simply to maximize his first period payoff, assuming correctly that he can have no effect on his second period payoff, or on the first period price. As the good is of no value in state 2 we may set $b_2 = 0$. Fig. 1 shows market clearance and price formation in states 1 and 2 in period 1.

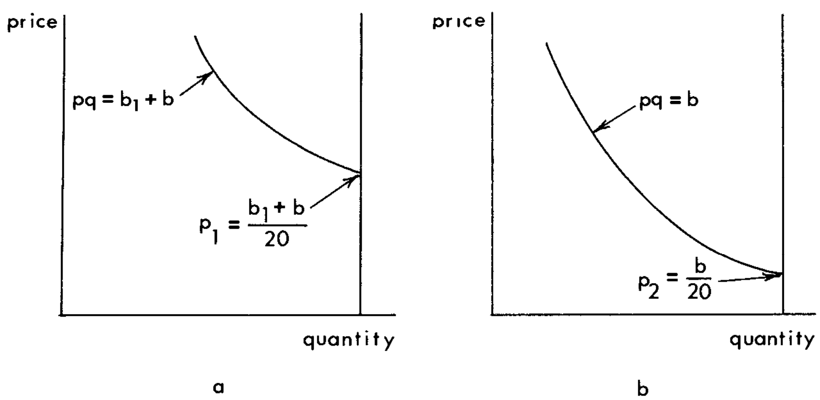


Fig. 1 a is state 1; b is state 2

The first period payoffs to the informed traders are

$$\Pi^I = \tfrac{1}{2}\left(A \log \frac{b_1}{p_1} + M - b_1\right) + \tfrac{1}{2}M = \tfrac{1}{2}\left(A \log \frac{b_1}{p_1} - b_1\right) + M. \tag{2}$$

The first period payoffs to the uninformed traders are

$$\Pi^U = \tfrac{1}{2}\left(A \log \frac{b}{p_1} + M - b\right) + \tfrac{1}{2}(M - b) = \tfrac{1}{2}A \log \frac{b}{p_1} - b + M, \tag{3}$$

where $p_1=(b_1+b)/20$, $p_2=b/20$ but each infinitesimal player treats p_1 and p_2 as fixed. Agent optimization gives

$$\frac{A}{p_1}=1 \quad \text{and} \quad \frac{A}{2b}=1. \tag{4), (5}$$

Suppose $A=B=10$. Then we have $b_1=10$, $b=5$, $p_1=3/4$ and $p_2=1/4$, $x_1^I=b_1/p_1=13\frac{1}{3}$, $x_1^U=b/p_1=6\frac{2}{3}$, $x_2^I=0$, $x_2^U=b/p_2=20$ and hence,

$$\Pi^I=5\log 13\tfrac{1}{3}-5+M, \tag{6}$$

$$\Pi^U=5\log 6\tfrac{2}{3}-5+M. \tag{7}$$

We may leave off the M to see the gains from trade.

In the second period both traders will be informed since $p_1=3/4\neq 1/4=p_2$. In that case their second price payoffs are

$$\tilde{\Pi}^I=\tilde{\Pi}^U=\tfrac{1}{2}\left(B\log\frac{\tilde{b}_1}{\tilde{p}_1}-\tilde{b}_1\right)+M-\tfrac{1}{2}\tilde{b}_2. \tag{8}$$

Of course if state 1 occurs in period 1, then by definition it occurs also in period 2. We find that $\tilde{b}_2=0$ for both traders and $\tilde{b}_1=10$, $p_1=1$, $p_2=0$, and

$$\tilde{\Pi}^I=\tilde{\Pi}^U=5\log 10-5+M. \tag{9}$$

Note again that the prices reveal the information, which is already known, anyway.

We have found the equilibrium outcomes and payoffs; we leave to the reader the full specification of the equilibrium strategies. It is clear that

$$\Pi^I+\tilde{\Pi}^I-(\Pi^U+\tilde{\Pi}^U)=5\log 2>0. \tag{10}$$

Observation 1. We have shown by example here that it is easy for a whole class of small traders to gain from extra information even though it is revealed by the prices formed.

Example 3. The purchase of information. Let us augment the game of Example 2 by introducing a period 0 in which type 2 players may pay Δ to learn Nature's move. (We shall assume that the players of type 1 know if the information has been bought or not.)

Let us consider two cases for the cost Δ of information:

$$\Delta<5\log 2, \qquad \Delta>5\log 2.$$

As we have seen, if no trader of type 2 purchases information, then the first period market yields a payoff of $5\log 13\frac{1}{3}-5+M$ to the informed and $5\log 6\frac{2}{3}-5+M$ to the uninformed. Regardless of whether information has been purchased, the second period payoff is $5\log 10-5+M$. If the cost Δ of information is greater than $5\log 2$, then this is a Nash equilibrium – no agent will purchase information. On the other hand, for $\Delta<5\log 2$ each agent will be tempted to purchase the information. Now suppose all but one of the continuum of type 2 agents has indeed purchased information. Each trader would then be earning $5\log 10-5+M$ in the first period. If the one remaining trader chose not to purchase the information, he would earn

$$\max_b \tfrac{1}{2}\left(10\log\frac{b}{1}-b\right)+M-\tfrac{1}{2}b. \tag{11}$$

Solving his first order condition $5/b=1$, we get that his expected utility is $5\log 5-5+M$, hence if $\Delta<5\log 2$ he will also purchase the information. If $\Delta<5\log 2$ the only (symmetric) equilibrium occurs where everybody purchases information. If $\Delta>5\log 2$ the only (symmetric) equilibrium occurs where nobody purchases information.

Let us now consider the game in which neither the type 1 player nor the type 2 player is initially informed, but each one can purchase information at a price $5\log 2<\Delta<10\log 2$. If nobody purchases information, then the payoffs in both periods will be the same: each player will act to

$$\max_b \tfrac{1}{2}\left(10\log\frac{b}{p}-b\right)+M-\tfrac{1}{2}b \tag{12}$$

giving the first order condition

$$\frac{5}{b}=1 \tag{13}$$

so that $b=5$, $p=1/2$ and the payoff to each player is $5\log 10-5+M$ in both periods. A player who purchased information could make

$$2\max_{b_1} \tfrac{1}{2}\left(10\log\frac{b_1}{1/2}-b_1\right)+M=2[5\log 20-5+M] \tag{14}$$

for a gain of $2[5\log 2]=10\log 2$. Now for $5\log 2<\Delta<10\log 2$ it is evident that every player will find it profitable to buy information; on the other hand once every player has purchased the information it is no longer so valuable (since it will then be revealed anyway in the second period). There is no N.E., in pure strategies. For $0<\Delta<5\log 2$ there is a Nash equilibrium in which every player purchases information.

Observation 2. If information is available for sale it may be bought at an N.E.; for sufficiently low but strictly positive prices it always will be by those agents who don't already know it. In contrast consider a R.E.E. with fully revealing prices. If we augment the R.E.E. model with a market for information, as above, there will be no incentive for any individual to purchase at this market. But then no private information will be available to be revealed by prices. This shows that R.E.E. do not exist with a market for information (as Grossman–Stiglitz pointed out). [Depending on the circumstances, there may be intermediate levels of the price of information for which the symmetric pure strategy N.E. in our game are also destroyed, as in Shubik (1984).]

Example 4

Observation 3. If the number of traders is finite it is possible that they will choose to conceal information in early markets if greater profits are to be made later. Thus the revelation of information is a phenomenon that attaches to perfect competition and may well break down in an oligopolistic setting.

To illustrate this, we consider a game with one player of each type. Player 1 can pick strategies which will disclose his information, and by doing so may make a higher payoff in the first period. He also has the choice of acting as though he were uninformed. By doing so he earns less in the first period but does not disclose information about the state of Nature.

Suppose for example we assume that there are two periods and the market structure and preferences in the first period are as in Example 2. In the second period the commodity for consumption in state 1 is 10 times as valued as in the first period; $A=10$, $B=100$. Furthermore the supply in each period is the same and the good cannot be inventoried by the consumers. The utility function of a trader can be written as

$$U^i=[\tfrac{1}{2}(10\log x_1^i+w_1^i)+\tfrac{1}{2}w_2^i]+[\tfrac{1}{2}(100\log \tilde{x}_1^i+\tilde{w}_1^i)+\tfrac{1}{2}\tilde{w}_2^i]=\Pi^i+\tilde{\Pi}^i. \tag{15}$$

First suppose there is only one period with one trader with information and one without. The payoffs are as follows:

$$\Pi^1=\tfrac{1}{2}\left\{10\log 20\frac{b_1}{b+b_1}-b_1\right\}+M, \tag{16}$$

$$\Pi^2=5\log 20\frac{b}{b+b_1}-b+M. \tag{17}$$

As there are only two traders they each influence price where $p_1=(b+b_1)/20$ and $p_2=b/20$.

We may construct table 1 showing the duopsony gains from trade solutions to the one period strategic market game.

Table 1
Four one state games: Duopsony payoffs.

	Type 2	
Type 1	Informed	Uninformed
Informed	9.013, 9.013	10.234, 7.643
Uninformed	7.643, 10.234	9.013, 9 013

A multiplying of these numbers by 10 yields the payoffs in the second period subgames. It is straightforward to observe that if the informed player chooses to earn 10.234 and thereby reveals his information in the first period, he will earn 9.013 in the second period. If, on the other hand, he plays as if he were uninformed in the first period the totals earned are 9.013 and 10.234. The full payoff with disclosure is 100.364 and without is 111.353. It is important to note that it does not matter that the player of type 2 knows that the other player lies; there is nothing he can do about it. He gets no information on Nature. This result is robust; we could have had k traders of each type, as long as each had influence on price. With a continuum of traders a single individual who is informed is tempted to save money by not buying worthless goods in state 2. All of them would do this, the price would change and the information would be signalled. In the following example we again assume a continuum of traders.

Example 5. Let us consider an economy in which every agent i has a utility $u^i(x,\tilde{x})$ of the form $u^i=u^i(x+\tilde{x})$. Each agent will be allowed to put up for sale however many goods he wants and to bid money for all the commodities. Inventorying will also be allowed. Since the form of the utility function implies that there is nothing to be gained by having a commodity today rather than waiting until period 2, one might suppose that the agents would wait until the first period prices had revealed all the information before doing nearly all their trading in period 2. In that case the final allocation would apparently be nearly the same as the fully revealing R.E.E. of the one period economy obtained by combining the endowments of the two time periods into one. However, we shall show not only that the Nash equilibrium allocation of the strategic market (bid–sell) game is different, but moreover we shall show that in our example it (ex ante) Pareto dominates the R.E.E.

allocation. This of course demonstrates the failure of the first welfare theorem for a R.E.E. allocation. It can be in the interest of all to trade before their information is complete.

Let there be two types of agents $\alpha \in (0,1]$, $\beta \in (1,2]$, two commodities (x and money) and 4 states of nature. Let each $\alpha \in (0,1]$ agent have utility:

$$U^{\alpha} = \tfrac{1}{4}\sqrt{x_1^{\alpha} + \tilde{x}_1^{\alpha}} + \tfrac{1}{4}\sqrt{(w_2^{\alpha} + \tilde{w}_2^{\alpha})\tfrac{1}{2}} + \tfrac{1}{4}\sqrt{w_3^{\alpha} + \tilde{w}_3^{\alpha}} + \tfrac{1}{4}\sqrt{x_4^{\alpha} + \tilde{x}_4^{\alpha}}. \tag{18}$$

For $\beta \in (1,2]$, let

$$U^{\beta} = \tfrac{1}{4}\sqrt{x_1^{\beta} + \tilde{x}_1^{\beta}} + \tfrac{1}{4}\sqrt{(w_2^{\beta} + \tilde{w}_2^{\beta})\tfrac{3}{2}} + \tfrac{1}{4}\sqrt{(w_3^{\beta} + \tilde{w}_3^{\beta})\tfrac{4}{3}} + \tfrac{1}{4}\sqrt{x_4^{\beta} + \tilde{x}_4^{\beta}}. \tag{19}$$

In states one and four, only the good has utility, in states two and three only money has utility. Let the agents of type 1 distinguish odd and even states, and let those of type 2 distinguish $s \in \{1,2\}$ from $s \in \{3,4\}$. (See table 2.)

Table 2

Type 1 distinguishes rows; type 2 columns.

	Type 2			
Type 1	$\bar{x}=20$		$\bar{x}=15$	
$\bar{M}=20$	1	x	3	M
$\bar{M}=30$	2	M	4	x

Let the endowment of type 1 agents be 20 units of money for $s \in \{1,3\}$ and 30 units of money for $s \in \{2,4\}$ in both periods and nothing else.

Let the endowment of type 2 agents be 20 units of x for $s \in \{1,2\}$ and 15 units of x for $s \in \{3,4\}$ in both periods, and nothing else.

It is easy to see that once the state of nature has been revealed, no further trade will take place. Hence we need only find (b_T, b_B), the money bids of a trader $\alpha \in (0,1]$ if he sees top or bottom respectively, and s_L, s_R, the amounts of x offered for sale by each $\beta \in (1,2]$ if he sees left or right, respectively. Thus $\alpha \in (0,1]$ acts to

$$\max_{b_T, b_B} \tfrac{1}{4}\sqrt{\frac{b_T}{p_1}} + \tfrac{1}{4}\sqrt{(60 - b_B)\tfrac{1}{2}} + \tfrac{1}{4}\sqrt{40 - b_T} + \tfrac{1}{4}\sqrt{\frac{b_B}{p_4}} \tag{20}$$

such that

$$0 \leqq b_T \leqq 20, \qquad 0 \leqq b_B \leqq 30$$

while each agent $\beta \in (1,2]$ acts to

$$\max_{s_L, s_R} \tfrac{1}{4}\sqrt{40-s_L}+\tfrac{1}{4}\sqrt{p_2 s_L \tfrac{3}{2}}+\tfrac{1}{4}\sqrt{p_3 s_R \tfrac{4}{3}}+\tfrac{1}{4}\sqrt{30-s_R} \tag{21}$$

such that

$$0 \leqq s_L \leqq 20, \qquad 0 \leqq s_R \leqq 15.$$

One can easily verify that $b_T=20$, $b_B=30$, $s_L=20$, $s_R=15$, $p_1=1$, $p_2=3/2$, $p_3=4/3$, $p_4=2$ comprise a Nash equilibrium, since for each $s \in \{1,2,3,4\}$, $b_s/s_s=p_s$ and the first order conditions

$$b_T=\frac{40}{1+p_1}, \qquad b_B=\frac{60}{1+\frac{1}{2}p_4},$$

$$s_L=\frac{40p_2}{\frac{3}{2}-p_2}, \qquad s_R=\frac{30p_3}{\frac{4}{3}+p_3}, \tag{22}$$

are satisfied. The prices are fully revealing, and in the second period there is no further trade. The expected utility of any type 1 agent is

$$U^\alpha=\tfrac{1}{4}\sqrt{20}+\tfrac{1}{4}\sqrt{15}+\tfrac{1}{4}\sqrt{20}+\tfrac{1}{4}\sqrt{15} \tag{23}$$

while for $\beta \in (1,2]$ we get

$$U^\beta=\tfrac{1}{4}\sqrt{20}+\tfrac{1}{4}\sqrt{45}+\tfrac{1}{4}\sqrt{80/3}+\tfrac{1}{4}\sqrt{15}. \tag{24}$$

By contrast, in the R.E.E. (where prices together with each agent's information fully reveal the state to him) we get

$$U^2=U^B=\tfrac{1}{4}\sqrt{40}+\tfrac{1}{4}\sqrt{30}. \tag{25}$$

8. Non-implementability of R.E.E.

We shall now show that if we replace the Shapley–Shubik mechanism with any weakly continuous, anonymous mechanism, then the N.E. approach remains distinct from R.E.E. To make this precise, suppose that each agent in $(0,N)$ has access to a set of moves Σ. All that we need assume about Σ is that it is a measurable space. The fact that each agent can use the same set Σ reflects the 'anonymity' of the mechanism, i.e., the market is blind to the private characteristics of the agents and cannot discriminate a priori among the moves permitted to them. Let $\tilde{\Sigma}$ denote the set of all measurable move-selections by the agents, i.e., $\tilde{\Sigma}$ is made up of all measurable functions from $(0,N]$ to Σ. A mechanism consists of two maps

$$\tilde{\Sigma} \xrightarrow{\pi} \mathbb{R}^L_+ \quad \text{and} \quad \Sigma \times \mathbb{R}^L_+ \xrightarrow{\phi} \mathbb{R}^L,$$

where

$$\int_0^N \phi(\tilde{\sigma}(t), \pi(\tilde{\sigma}))\mathrm{d}t = 0 \quad \text{for all } \tilde{\sigma} \in \tilde{\Sigma}$$

and

$$\phi(\sigma, p) \cdot p = 0 \quad \text{for all } \sigma \in \Sigma, \quad p \in \mathbb{R}^L_+.$$

Let us interpret our symbols: $\pi(\tilde{\sigma})$ is the price vector produced when agents move according to $\tilde{\sigma}$, and $\phi(\sigma,p)$ is the net trade that accrues to any trader if his move is $\sigma \in \Sigma$ and the prices p prevail. Note that the absence of a superscript of t on ϕ, and the common access to Σ by each t discussed earlier, makes the mechanism *anonymous* (we could also add that π depends only on the distribution induced by $\tilde{\sigma}$ on Σ though we don't need this) and requires trade to be conducted via prices.[16] For the mechanism (Σ, π, ϕ) let us construct the game tree exactly as was done in the case of the Shapley–Shubik mechanism. For ease of discussion consider only one time period. Then the strategy of an agent of type n is a map σ_* from the state space to moves, $S \xrightarrow{\sigma_*} \Sigma$, which is compatible with the partition I^n, i.e., constant on each piece of it. Let Σ^n_* denote the set of all strategies of an agent of type n; and let $\tilde{\Sigma}_* =$ the set of all measurable strategy-selections by the agents. A choice $\tilde{\sigma}_* \in \tilde{\Sigma}_*$ by agents $t \in (0,N]$ [where $\tilde{\sigma}_*(t) \in \Sigma^n_*$ if $t \in (n-1,n]$] produces a reallocation $x(\tilde{\sigma}_*) = (0,N] \to \mathbb{R}^{L \times S}$ and prices $p(\tilde{\sigma}_*) \in \mathbb{R}^{L \times S}_+$. This is done in the obvious manner via the mechanism. For any state of nature $s \in S$, the strategy $\tilde{\sigma}_*(t)$ of each agent t prescribes a move $(\tilde{\sigma}_*(t))(s)$ by him. Given the collection of moves by all the agents, trades and prices occur in state s according to ϕ and π.

We now proceed to define an N.E. of this anonymous game. For $\sigma(t) \in \Sigma^n_*$ and $\tilde{\sigma}_* \in \tilde{\Sigma}_*$, denote by $(\tilde{\sigma}_* | \sigma(t))$ the same choice as $\tilde{\sigma}_*$ except that $\tilde{\sigma}_*(t)$ is replaced by $\sigma(t)$. We will say that $\tilde{\sigma}_*$ is a Nash equilibrium if, for each $t \in (n-1,n]$ and each $n \in N$,

$$x^t(\tilde{\sigma}_*) \quad \text{maximizes} \quad u^n(x + e^n) \quad \text{on}$$

$$x \in \{x^t(\tilde{\sigma}_* | \sigma(t)) : \sigma(t) \in \Sigma^n_* \text{ and } x^t(\tilde{\sigma}_* | \sigma(t)) + e^n \in \mathbb{R}^{L \times S}_+\}.$$

[16] We can allow for the possibility that when no trade occurs, no price need be quoted. Simply extend the range of π to $\mathbb{R}^L_+ \cup \{h\}$ and require that $\phi(\sigma, h) = 0$ for all $\sigma \in \Sigma$.

Note that the set of strategies to which an agent can deviate depends upon what others have chosen. Without this stipulation it may happen that $x^t(\tilde{\sigma}_* | \sigma(t)) + e^n \notin \mathbb{R}_+^{L\times S}$, i.e., agent t may be assigned a net trade which he cannot honor. Thus we are here talking of a 'generalized game' as in Debreu (1952).

First we will show that, if we impose a weak continuity requirement on ϕ and π, then one can always find an open set of economies, each of which has a R.E.E. that is not an N.E. In short *there does not exist any continuous mechanism which implements R.E.E. everywhere.*

To define continuity of π we will need a little notation. Let $\tilde{\sigma}:(0,N]\to\Sigma$ and $\tilde{\tilde{\sigma}}:(0,N]\to\Sigma$ be two move selections. For $0<\lambda\leq N$, let $\lambda\tilde{\sigma}+(N-\lambda)\tilde{\tilde{\sigma}}$ denote the move selection in which

$$t \text{ chooses} \begin{cases} \tilde{\sigma}(t) & \text{if} \quad t\in(0,\lambda], \\ \tilde{\tilde{\sigma}}(t) & \text{if} \quad t\in(\lambda,N]. \end{cases}$$

Continuity assumption.

(i) $\pi(\lambda\tilde{\sigma}+(M-\lambda)\tilde{\tilde{\sigma}})$ is continuous in λ,

(ii) $\phi(\sigma,p)$ is continuous in p for all $\sigma\in\Sigma$, $p\in\mathbb{R}_+^L$.[17]

Fix S, L, N and $\{I^n : n\in N\}$ with $I^n\neq I^{n'}$ for some pair n, n'. Let $\mathscr{E}\times\mathscr{U}^N$ be the set of all endowments and utilities (satisfying the assumptions of section 6) for this data, i.e., $\mathscr{E}\times\mathscr{U}^N=\{e^1,\ldots,e^N;\ u^1,\ldots,u^N : e^n \varepsilon \mathbb{R}_+^{L\times S},\ u^n:\mathbb{R}_+^{L\times S}\to R\}$. Recall that $\mathscr{E}$ is a topological space (with the C^2-norm on utilities).

Theorem 2. Given any anonymous and continuous mechanism, there exists an open set $\mathscr{V}$ of economies in $\mathscr{E}\times\mathscr{U}^N$ such that each economy in $\mathscr{V}$ has a (type-symmetric) R.E.E. that is infeasible in the mechanism [i.e., no choice of (type-symmetric) strategies can lead to it]. In particular, these R.E.E.'s cannot be implemented as (type-symmetric) N.E.'s of the mechanism.

Proof. First, for simplicity, we take $S=\{1,2\}$, $L=\{1,2\}$, $N=\{1,2\}$, $I^1=(\{1,2\})$, $I^2=(\{1\},\{2\})$. It will be clear that our argument can be embedded in any general model with asymmetric information.

Suppose (Σ,ϕ,π) is an anonymous, continuous mechanism and $\mathscr{D}\subset\mathscr{E}\times\mathscr{U}^N$ is a dense set on which R.E.E. are feasible via the mechanism. We will show that this leads to a contradiction.

For any type-symmetric R.E.E. of an economy in $\mathscr{E}\times\mathscr{U}^N$, define its 'profile' to be the table below (where the price of commodity 2 is fixed at unity):

[17]Note that (i) and (ii) together imply that (iii) the net allocation to trader t, $\phi(\tilde{\sigma}_t,\pi(\lambda\tilde{\sigma}+(N-\lambda)\tilde{\tilde{\sigma}})$ is continuous in λ, if $t<\lambda$. If we extend π to include the no trade point h, then we need to impose (iii) as well as (i) and (ii) in order to prove Theorem 2

State	1	2
Price of commodity 1	p^1	p^2
Net trade of commodity 1 by agents of type 1	z^1	z^2

A key fact that we will exploit is that as we vary over $\mathscr{D}$, the set of profiles p^1, p^2, z^1, z^2 obtained is dense in $\mathbb{R}^2_+ \times \mathbb{R}^2$. This follows at once from the well-known fact that

(i) R.E.E. vary continuously on an open dense set $\mathscr{W}$ of $\mathscr{E} \times \mathscr{U}^N$,
(ii) the profiles obtained from R.E.E. of economies in $\mathscr{W}$ are dense in $\mathbb{R}^2_+ \times \mathbb{R}^2$.

It is clear that we can find an economy in $\mathscr{D}$ which has an R.E.E. profile

1	2
p^1	p^2
z^1	z^2

with $p^1 > p^2$. By assumption this R.E.E. can be achieved through type-symmetric strategies in the game tree built out of (Σ, ϕ, π). Denote them:

State	1	2
Moves of type 1	σ	σ
Moves of type 2	σ^1	σ^2

We claim that there exists another economy in $\mathscr{D}$ with an R.E.E. that has:

(*I*)

State	1	2
Price of good 1	$\bar{p}^1$	$\bar{p}^2$
Final holding of good 1 by type 1	$\bar{z}^1$	$\bar{z}^2$

(*II*) $p^1 > \bar{p}^1 > \bar{p}^2 > p^2$, and

(*III*) one of the following four cases obtaines:

Case A.

(i) $\phi(\sigma, \bar{p}^1) + \phi(\sigma^1, \bar{p}^1) \geqq 0$,
(ii) $\phi(\sigma, \bar{p}^2) + \phi(\sigma^2, \bar{p}^2) \geqq 0$,
(iii) $\bar{z}^1 > \phi(\sigma^1, \bar{p}^1)$,
(iv) $\bar{z}^2 > \phi(\sigma^2, \bar{p}^2)$.

Case B. $\geqq$ in (i),
$<$ in (ii),
$>$ in (iii),
$<$ in (iv).

Case C. $<$ in (i),
$\geqq$ in (ii),
$<$ in (iii),
$>$ in (iv).

Case D. $<$ in (i),
$<$ in (ii),
$<$ in (iii),
$<$ in (iv).

Note that the four cases correspond to all the possible combinations of inequalities in (i) and (ii). Given any such case, it is always possible to choose numbers $\bar{z}^1$ and $\bar{z}^2$ satisfying (iii) and (iv).

The claim follows straightforwardly from the continuity of ϕ in prices, and the fact that profiles of $\mathscr{D}$ are dense in $\mathbb{R}^2_+ \times \mathbb{R}^2$. We leave it to the reader.[18]

Suppose Case A of (III) holds. Let $\bar{\sigma}$ denote the move of type 1 of a strategy-choice which yields $(\bar{p}^1, \bar{p}^2, \bar{z}^1, \bar{z}^2)$. Put

$$p^* = \pi(\sigma, \bar{\sigma}).$$

Here $(\sigma, \bar{\sigma})$ stands for the function from $(0,2]$ to Σ which maps t to σ if $t \in (0,1]$ and to $\bar{\sigma}$ if $t \in (1,2]$. Now at least one of the two cases must occur:

(a) $\bar{p}^1 \in (p^*, p^1)$,
(b) $\bar{p}^2 \in (p^2, p^*)$.

If (a) occurs, consider $\lambda\sigma^1 + (1-\lambda)\bar{\sigma}$, i.e., the choice of moves by agents of type 2 ($t \in (1,2]$) at which

$$t \text{ chooses } \sigma^1 \quad \text{if} \quad t \in (1, 1+\lambda],$$

$$t \text{ chooses } \bar{\sigma} \quad \text{if} \quad t \in (1+\lambda, 2].$$

(Here $0 \leqq \lambda \leqq 1$.) Then

$$\pi(\sigma, \lambda\sigma^1 + (1-\lambda)\bar{\sigma}) = p^1 \quad \text{if} \quad \lambda = 1,$$

$$= p^* \quad \text{if} \quad \lambda = 0.$$

[18]We may also assume that $\phi(\sigma, p_1) \neq 0$ and $\phi(\sigma, p_2) \neq 0$, and (from the continuity of ϕ in p) we may assume that $\bar{p}_i$ is chosen close enough to p_i that $\phi(\sigma, p)$ is bounded away from 0 on $[\bar{p}_1, p_1] \cup [\bar{p}_2, p_2]$.

By continuity of π in λ, we must have

$$\pi(\sigma, \bar{\lambda}\sigma^1 + (1-\bar{\lambda})\bar{\sigma}) = \bar{p}^1$$

for some $0 < \bar{\lambda} < 1$.[19] We will show that at this choice of strategies, the total net trade will not balance, i.e., integrate to zero. The total net trade is

$$\phi(\sigma, \bar{p}^1) + \bar{\lambda}\phi(\sigma^1, \bar{p}^1) + (1-\bar{\lambda})\phi(\bar{\sigma}, \bar{p}^1) = \phi(\sigma, \bar{p}^1) + \bar{\lambda}\phi(\bar{\sigma}^1, \bar{p}^1) + (1-\bar{\lambda})\bar{z}^1,$$

and this is strictly positive since $\bar{z}^1 > \phi(\sigma^1, \bar{p}^1)$ and $\phi(\sigma, \bar{p}^1) + \phi(\sigma^1, \bar{p}^1) \geqq 0$ (recall Case A), a contradiction. If (b) holds a similar contradiction is reached. Cases B, C, D are worked out just like Case A. Q.E.D.

Remark. Theorem 2 implies in particular that the excess demand game does not continuously implement R.E.E., even if the economies are restricted to the generic set where the R.E.E. correspondence is well-behaved. The reason is that for non-pathological economies, the R.E.E. allocation for some commodity may be increasing in its own price. To implement such an R.E.E. would require the submission of both upward and downward sloping excess demands. But this gives rise to multiple market clearing prices. No selection rule can continuously specify a single price. Even if we drop the requirement that a price is quoted when no trade is allowed, so that the excess demand mechanism could specify no trade at 'h' when there are multiple market clearing prices, the mechanism would not be continuous in its allocation.

Remark. It is natural to ask if a stronger result is possible, namely whether for any anonymous, and continuous mechanism there is also a dense set of economies in $\mathscr{E} \times U^N$ on which R.E.E. cannot be implemented. Consider the two trader, two state, two commodity case in which the move space Σ consists of all linear, *downward* sloping excess demand functions $d(p)$ for good 1 (the quantity named for the other good is dictated by budget balancing and $p_2 = 1$). The map π is determined by the necessarily unique intersection of aggregate excess demand with the price axis; ϕ gives each trader the trade he has submitted at this market clearing price. Note that Σ is finite-dimensional and the mechanism is anonymous and continuous. It is clear that there is an open set of economies in $\mathscr{E} \times U^2$ whose R.E.E. profiles all satisfy the property that $p^1 > p^2$ and $z^1 < z^2$. One can easily check that the R.E.E.'s coincide with the N.E.'s of the generalized game, for this open set of economies.

[19] This remains true even when the range of π is extended to include the no trade point h Recall that $\phi(\sigma, p)$ is bounded away from 0 for $p \in [\bar{p}_1, p_1]$, and apply continuity axiom (iii).

9. Proof of Theorem 1

The proof consists of three steps. We will define a 'potential Nash Equilibrium' (p.N.E.) which exists for every $\Gamma=(e,\tilde{e},u)\in\mathscr{E}\times\mathscr{U}^N$ (section 9.1). Then we define $\mathscr{V}$ and prove that for all $\Gamma\in\mathscr{V}$ the set of p.N.E. of Γ is finite (section 9.2). Finally we show that for $\Gamma\in\mathscr{V}$, the set of p.N.E. = the set of N.E. = the set of fully revealing N.E. (section 9.3). Throughout we cut the notation in half by assuming that the agents always put all their endowments up for sale (the 'sell-all' model). The proof is nearly identical in the buy–sell case.

9.1. Potential Nash equilibrium

Fix $\Gamma=(e,\tilde{e},u)$ in $\mathscr{E}\times\mathscr{U}^N$. The *fictitious game* Γ^* is obtained from Γ by the modifications: (i) the information partition $\tilde{I}^n$ of each type in period 2 is replaced by $I^1\vee\cdots\vee I^N$. [W.l.o.g. assume that $I^1\vee\cdots\vee I^N=I^*=(\{1\},\ldots,\{S\})$ from now on.] (ii) Strategies are restricted to be bids contingent *only* on the information about chance moves and not contingent, beyond this, on others' moves, i.e., $\tilde{b}^n(s,z)=\tilde{b}^n(s)$ for all $n\in(0,N]$.

For $\Delta>0$ consider the *Δ-modified fictitious game* Γ^*_Δ in which [in addition to (i) and (ii)] an external agency is imagined to have placed bids of size Δ in each of the $2(L-1)S$ trading posts. This does not affect the strategy sets of Γ^* but only the strategy-to-outcome map.

A *potential Nash Equilibrium* (p.N.E.) of Γ is simply an N.E. of Γ^*. If $\eta(\Gamma)$ denotes the set of N.E. of Γ, then clearly p.N.E. of $\Gamma=\eta(\Gamma^*_0)$.

Let $\cdot\Sigma^n$ denote the strategy-set of type n in the game Γ^*_Δ, $\Delta\geqq 0$. A typical element of Σ^n consists of a pair of vectors b^n, $\tilde{b}^n$ in $\mathbb{R}_+^{(L-1)\times S}$ measurable w.r.t. I^n, I^* respectively. Since utilities are *strictly* concave, and the set of agents $(0,N]$ is non-atomic, it is obvious that at any N.E. of Γ^*_Δ agents of a given type use the same strategy. Therefore in our analysis of $\eta(\Gamma^*_\Delta)$ we may restrict ourselves to the set $\Sigma=\Sigma^1\times\cdots\times\Sigma^N$.

For $\mu>0$ denote by Σ_μ the subset of Σ at which all prices p_{ls}, $\tilde{p}_{ls}$ $(l\in L-1,\ s\in S)$ in the two periods are at least μ.

Lemma 1. There is a $\mu>0$ such that if $\Gamma\in\mathscr{E}\times\mathscr{U}^N$ then $\eta(\Gamma^*_\Delta)\subset\Sigma_\mu$ for $\Delta>0$.

Proof. First let us show that there is a μ_1 such that if the first period moves at some N.E. of Γ^*_Δ are b, then $p^b_{ls}>\mu_1$ for all s,l. (By p^b_{ls} we mean the first-period prices that accrue from b in the game Γ^*_Δ.) Note that since $\Delta>0$, we must have $p^b\gg 0$.

Case 1

$$\sum_{l\in L-1} b^n_{ls}<e^n_{Ls}$$

for some $n \in N$ and $s \in S$. If an agent of type n increases his bid $b^n_{lr}(1)$ $(r \in I_n(s))$ by $\varepsilon > 0$ then the increase in his payoff, for small ε, is approximately

$$\varepsilon\left[\sum_{r \in I_n(s)}\left(\frac{\partial u^n}{\partial x_{lr}}\bigg/ p^b_{lr} - \frac{\partial u^n}{\partial u_{Lr}}\right)\right] \geqq \varepsilon[\sigma/p^b_{ls} - |I_n(s)|\sigma']$$

for any $r \in I_n(s)$. This must be non-positive, therefore

$$\boxed{p^b_{ls} \geqq \sigma/(|I_n(s)| \cdot \sigma') \geqq \sigma/S\sigma'}.$$

The same argument shows that for any l, if $b^n_{ls} > 0$, then for any $l' \in L-1$, $p^b_{l's} \geqq p^b_{ls}\sigma/S\sigma'$, since agent n can always withdraw ε from market l and bid it on market l' instead.

Case 2

$$\sum_{l \in L-1} b^n_{ls} = e^n_{Ls} \quad \text{for all } n \in N, \quad \text{and some } s \in S.$$

Let $\bar{e}_{ls} = \sum_{n \in N} e^n_{ls}$. Then if Case 2 holds, for at least some $l \in L-1$, $p^b_{ls} \geqq \bar{e}_{Ls}/\bar{e}_{ls}(L-1)$, since by hypothesis all of $\bar{e}_{Ls}$ is bid and it must be bid on $L-1$ markets. But now by the argument after Case 1, we get that for all $l' \in L-1$, $p^b_{l's} \geqq (\bar{e}_{Ls})/(\bar{e}_{ls}(L-1)S\sigma')$. Letting $\mu_1 = \min_{l \in L-1, s \in S}\{1, \text{less } \bar{e}_{Ls}/\bar{e}_{ls}(L-1)\}(\sigma/S\sigma')$, we have shown: $b \in \eta(\Gamma^*_\Delta) \Rightarrow p^b_{ls} > \mu_1$ for all l and s. In an exactly analogous manner, one can check that there is a $\mu_2 > 0$ such that if $\tilde{b}$ are the second-period moves at any N.E. of Γ^*_Δ then $\tilde{p}^b_{ls}(2) > \mu_2$ for all l and s. Then, with $\mu = \min\{\mu_1, \mu_2\}$, the lemma follows (recall the bounds on endowments in $\mathscr{E}$). Q.E.D.

Lemma 2. If $\Delta > 0$, then $\eta(\Gamma^*_\Delta)$ is non-empty for any $\Gamma \in \mathscr{E} \times \mathscr{U}^N$.

Proof. If $\Delta > 0$ the strategies-to-outcome map is continuous. (It blows up if $\Delta = 0$, i.e., in the unmodified fictitious game Γ^*_0 at strategies which produce a zero price in any trading post ... hence the importance of Lemma 1.) The proof now involves a straightforward use of Kakutani's fixed point theorem. Q.E.D.

Lemma 3. $\eta(\Gamma^*)$ is non-empty for any $\Gamma \in \mathscr{E} \times \mathscr{U}^N$.

Proof. Take a sequence $\{\Delta^m\}$, $\Delta^m \to 0_+$. Let ${}^m b$, ${}^m\tilde{b} \in \eta(\Gamma_{\Delta^m})$. (By Lemma 2 such ${}^m b$, ${}^m\tilde{b}$ exist.) Let ${}^*b$, ${}^*\tilde{b}$ be a cluster-point of the $\{{}^m b, {}^m\tilde{b}\}$. By Lemma 1, p^{*b}_{ls}, $p^{*\tilde{b}}_{ls} > \mu > 0$ for $l \in L-1$ and $s \in S$. Then ${}^*b$, ${}^*\tilde{b}$ is a point of continuity of the payoff functions, from which it easily follows that ${}^*b, {}^*\tilde{b} \in \eta(\Gamma^*)$. Q.E.D.

Remark. A straightforward fixed-point argument was not possible because of the singularity of the strategies-to-outcome map at places which produced zero prices. This made the Δ-approximation necessary.

9.2. Generic full revelation by prices

The idea of the proof is very simple; the execution is somewhat more complicated because of the possibility that a player may choose to be at a vertex of his strategy set, thus preventing us from differentiating in all directions. Ignoring that possibility for now, consider the set of strategy vectors $T=\prod_{n\in N}\sum_{-\varepsilon}^{0}\cap\sum_{\frac{1}{2}\mu}^{0}$, that is vectors $(b_{is}^n, \tilde{b}_{is}^n)$, $n\in N$, $s\in S$, $i=1,\dots,L-1$ satisfying b^n is measurable with respect to I^n,

$$b_{is}^n, \tilde{b}_{is}^n > -\varepsilon, \qquad \sum_{i=1}^{L-1} b_{is}^n < e_{Ls}^n+\varepsilon,$$

$$\sum_{i=1}^{L-1} \tilde{b}_{is}^n < \tilde{e}_{Ls}^n+\varepsilon \quad \text{for } i=1,\dots,L-1, \quad s\in S,$$

and lastly that the resulting prices $p_{is}\equiv\sum_{n\in N} b_{is}/\sum_{n\in N} e_{is}$ and $\tilde{p}_{is}=\sum_{n\in N}\tilde{b}_{is}/\sum_{n\in N}\tilde{e}_{is}$ are all strictly greater than $\frac{1}{2}\mu>0$.[20] Note that T is a manifold of dimension $\sum_{n\in N}(\#I^n+\#S)(L-1)\equiv k$. These strategies form a 'non-vertex', potential Nash equilibrium if and only if

(1) $(b^n,\tilde{b}^n)$ is in the compact set $\sum_\mu$ of bids that give rise to prices, greater or equal to $\mu>0$,

(2) for each $r\in I^n$, and each $i=1,\dots,L-1$,

$$\Phi_{ri}^n=\sum_{s\in r}\left[\frac{\partial U^n}{\partial X_{is}}\left(\sum_{n\in N} e_{is}^n\Big/\sum_{n\in N} b_{is}^n\right)-\frac{\partial u^n}{\partial X_{Ls}}\right]=0, \quad \text{and}$$

(3) for each $s\in S$, and $i=1,\dots,L-1$,

$$\tilde{\Phi}_{si}^n=\frac{\partial U^n}{\partial \tilde{X}_{is}}\left(\sum_{n\in N}\tilde{e}_{is}^n\Big/\sum_{n\in N}\tilde{b}_{is}^n\right)-\frac{\partial u^n}{\partial \tilde{X}_{Ls}}=0,$$

where $\partial u^n/\partial X$, $\partial u^n/\partial\tilde{X}$ are evaluated at $X_{is}^n=e_{is}^n+b_{is}/p_{is}$,

$$\tilde{X}_{is}^n=\tilde{e}_{is}^n+\frac{\tilde{b}_{is}^n}{\tilde{p}_{is}} \quad \text{for } i=1,\dots,L-1, \quad s\in S$$

[20] Of course some x_{is}^n may now end up negative (since T allows for negative bids) but if ε is small the consumption bundle $x,\tilde{x}$ will still lie in the neighborhood η on which u^n is defined. Note also that we have assumed, as in the last section that $I^{n2}=I^*=S$ so that there is no measurability restriction on $\tilde{b}^n$

$$X_{Ls}=e^n_{Ls}-\sum_{i=1}^{L-1} b_{is}+\sum_{i=1}^{L-1} p_{is}e^n_{is},$$

$$\tilde{X}_{Ls}=\tilde{e}^n_{Ls}-\sum_{i=1}^{L-1} \tilde{b}_{is}+\sum_{i=1}^{L-1} \tilde{p}_{is}\tilde{e}^n_{is}, \quad s\in S.$$

Notice that we have $\sum_{n\in N}(\# I^n+\# S)(L-1)$ equations to satisfy in equilibrium and exactly the same number of unknowns.

Recall that our definition of the utility space $\mathscr{U}$ implied that if $u^N\in\mathscr{U}$ then adding a sufficiently small linear function $(c,\tilde{c})\cdot(x,\tilde{x})$ to u^N produces a utility also in $\mathscr{U}$. Hence we can perturb the equilibrium equations locally in any way we like by perturbing the utilities. In other words, the map

$$\psi:\mathscr{U}^N\times T\to\mathbb{R}^{\sum_{n\in N}(\# I^n+S)(L-1)}\equiv\mathbb{R}^k$$

given by $\psi(u,b)=\prod_{n\in N}(\Phi^n\times\tilde{\Phi}^n)$ is transverse to $0\in\mathbb{R}^k$. Hence by the transversality theorem[21] there is an open and dense subset of $\mathscr{U}^N$ for which there can be only a finite number of 'non-vertex' equilibria. Furthermore, if $s\neq s'$ there is some agent n such that $r\neq r'\in I^n$, $s\in r$, $s'\in r'$. Hence the map $\bar{\psi}:\mathscr{U}^N\times T\to\mathbb{R}^{k+1}$ given by

$$\bar{\psi}(u,b)=\left(\psi(u,b),\left(\sum_{n\in N} b^n_{1s}\Big/\sum_{n\in N} e^n_{1s}\right)-\left(\sum_{n\in N} b^n_{1s'}\Big/\sum_{n\in N} e^n_{1s'}\right)\right)$$

is also transverse to $0\in\mathbb{R}^{k+1}$. Hence for an open and dense subset of $\mathscr{U}^N$ there are a finite number of non-vertex equilibria in all of which price p_1 alone distinguishes between s and s'.[22] By proceeding in a straightforward manner for all pairs s and s' in S we get our main proposition for non-vertex equilibria.

We can easily take account of the cases where, in equilibrium some $b^n_{is}=0$ and the first order condition (2) is a strict inequality instead of equality. Simply fix $b^n_{is}=0=b^n_{is'}$, for $I^n(s)=I^n(s')$, and drop the corresponding equation in ψ. We then get a map with one less variable and one less equation, and the same logic applies. In fact the finite set $\psi_{\bar{u}}^{-1}(0)$, for fixed $\bar{u}$, may now include some strategies where the b^n_{is} first order condition is strictly violated, so that we have shown even more than we need to.

[21]Recall that ψ $\mathscr{U}^N\times T\to\mathbb{R}^k$ is transverse to 0 if the Jacobian $D\psi_{\bar{u},\bar{b}}$ has full rank at any $(\bar{u},\bar{b})$ at which $\psi(\bar{u},\bar{b})=0$. We have just noted that even restricted to derivatives with respect to u alone $D\psi$ has full rank. Recall the transversality theorem, which concludes that if $\Sigma_\mu\subset T$ is compact, then there is an open and dense subset $\mathscr{V}$ of $\mathscr{U}^N$ in which $D\psi_{\bar{u},\bar{b}}$ has full rank when restricted to the b derivatives, for any $\bar{u}\in\mathscr{D}$, $\bar{b}\in\Sigma_\mu$, with $\psi(\bar{u},\bar{b})=0$

[22]If $\bar{\psi}.\mathscr{U}^N\times T\to\mathbb{R}^{k+1}$ is transverse to 0, then there is some open, dense subset $\tilde{\mathscr{V}}\subset\mathscr{U}^N$ for which $D\bar{\psi}_{\bar{u},\bar{b}}$ has full rank $(k+1)$ when restricted to the b derivatives, for any $\bar{u}\in\tilde{\mathscr{V}}$, $\bar{b}\in\Sigma_\mu$, with $\bar{\psi}(\bar{u},\bar{b})=0$. Since this is impossible, it follows that there is no $\bar{b}\in\Sigma_\mu$ such that $\bar{\psi}(\bar{u},\bar{b})=0$.

On the other hand, if in some potential Nash equilibrium all the $b^n_{is}=0$, or if $b^n_{is}=e^n_{is}$, so that player n is at a vertex of his strategy set bidding all his money on one commodity i or bidding none of it, and if n is the only player type that distinguishes s from s' then if we drop all the b^n_{is} strategies, setting $b^n_{is}=e^n_{Ls}$ and all the others equal to 0, then there is no guarantee that the map $\bar{\psi}$ is transverse to 0 (although ψ still will be). In other words, even if we perturb u^N slightly, player n may still set $b^n_{is}=e^n_{Ls}$, and it may happen that $p_{is}=p_{is'}$ for all $i=1,\dots,L-1$. This problem is eliminated by perturbing the endowments e^n_{is}, $e^n_{is'}$. The details are obvious and left to the reader. We now consider maps of the form $\bar{\psi}:\mathscr{E}\times\mathscr{U}^N\times T\rightarrow\mathbb{R}^{k+1}$ and obtain an open, dense set $\mathscr{V}$ of full measure in $\mathscr{E}\times\mathscr{U}^N$.

9.3. Completion of the proof

It remains to check that the set of p.N.E. of $\Gamma=$N.E. of Γ. From the fact that prices are fully revealing for $\Gamma\in\mathscr{V}$ it follows that $\tilde{I}^n$ is indeed equivalent to I^*, and so p.N.E.(Γ)$\subset$N.E.(Γ) for $\Gamma\in\mathscr{V}$. To show the reverse inequality we must repeat the whole argument for each of the finite number of information levels $I^{n2}/n\in N$ between I^n and I^* for each trader, requiring that $\bar{b}^n$ be measurable with respect to I^{n2}. In each case we will get an open dense set $\mathscr{V}'$ in which prices reveal I^* and agents should not be constrained to $I^{n2}\neq I^*$. Taking the finite intersection of open dense sets given a new open and dense $\mathscr{V}$. For this redefined $\mathscr{V}$, $\Gamma\in\mathscr{V}$ implies p.N.E.(Γ)$\supset$N.E.(Γ). Recall that in the introduction we noted the proof in Dubey and Kaneko (1982) that for non-atomic Γ there is no loss of generality in taking $\bar{b}^n$ to be defined on S, measurable with respect to some I^{n2}.

References

Abraham, R. and J. Robbin, 1967, Transversal mappings and flows (Benjamin, New York).
Allen, B., 1981, Generic existence of completely revealing equilibria for economies with uncertainty when prices convey information, Econometrica, 1173–1199.
Allen, B, 1982, Approximate equilibria in microeconomic rational expectations models, Journal of Economic Theory 26, 244–260
Anderson, R and H Sonnenschein, 1982, On the existence of rational expectations equilibrium, Journal of Economic Theory 26, 261–278.
Blume, L and D Easely, 1982, Learning to be rational, Journal of Economic Theory 26, 340–351.
Bray, M, 1982, Learning, estimation, and the stability of rational expectations, Journal of Economic Theory 26, 318–339
Bray, M and D Kreps, 1981, Rational learning and rational expectations, Discussion paper (Stanford University, Stanford, CA)
Debreu, G., 1952, A social equilibrium existence theorem, Proceedings of the National Academy of Sciences of the USA 38, 886–893
Dubey, P., 1982, Price quantity strategic market games, Econometrica 50, no. 1, 111–126.
Dubey, P. and M. Kaneko, 1982, Information about moves in extensive games I, Cowles Foundation discussion paper no 625 (Yale University, New Haven, CT).

Dubey, P. and J.D Rogawski, 1982, Inefficiency of Nash equilibria in a private goods economy, Cowles Foundation discussion paper no 632 (Yale University, New Haven, CT)

Dubey, P. and L S. Shapley, 1980, Noncooperative exchange with a continuum of traders Two models. Technical report (Institute for Advanced Studies. Hebrew University, Jerusalem).

Dubey, P and M Shubik, 1978, The noncooperative equilibria of a closed trading economy with market supply and bidding strategies, Journal of Economic Theory 17, no 1, Feb, 1–20.

Dubey, P. and M Shubik, 1980, A strategic market game with price and quantity strategies. Zeitschrift fur Naturalekonomie, 24–34

Dubey, P, A Mas-Colell and M. Shubik, 1980, Efficiency properties of strategic market games. An axiomatic approach, Journal of Economic Theory 22, no. 2

Green, J, 1973, Information, efficiency, and equilibrium, Discussion paper no. 284 (Harvard University, Cambridge, MA).

Grossman, S.J., 1976, On the efficiency of competitive stock markets where agents have diverse information, Journal of Finance 31, 573–585

Grossman, S J, 1977, The existence of future markets, noisy rational expectations, and informational externalities, Review of Economic Studies 64, 431–449

Grossman, S J, 1978, Further results on the informational efficiency of competitive stock markets, Journal of Economic Theory 18, 81–101

Grossman, S.J., 1981, An introduction to the theory of rational expectations under asymmetric information, Review of Economic Studies, 541–560

Grossman, S J and J K Stiglitz, 1980. On the impossibility of informationally efficient markets, American Economic Review 70, 343–400

Hayek, F, 1937, 'Economics and knowledge' and 'The use of knowledge in society', Reprinted in Individualism and economic order, 1980 (Midway, Chicago, IL).

Hellwig, M, 1982, Rational expectations equilibrium with conditioning on past prices. A mean variance example, Journal of Economic Theory 26, 279–312

Jordan, J, 1982a, The generic existence of rational expectations equilibrium in the higher dimensional case, Journal of Economic Theory 26, 224–243.

Jordan, J, 1982b, A dynamic model of expectations equilibrium, Journal of Economic Theory 28, 235–254

Lucas, R E., 1972, Expectations and the neutrality of money, Journal of Economic Theory, 103–124

Milgrom, P, 1981, Rational expectations, information acquisition, and competitive bidding, Econometrica, 291–943.

Muth, J, 1961, Rational expectations and the theory of price movements, Econometrica, 315–335.

Radner, R, 1979, Rational expectations equilibrium Generic existence and the information revealed by prices, Econometrica 47, 655–678

Shapley, L.S and M Shubik, 1977, Trade using one commodity as a means of payment, Journal of Political Economy 85.

Shubik, M, 1973, Commodity money, oligopoly, credit and bankruptcy in a general equilibrium model, Western Economic Journal 11

Shubik, M, 1984, On the value of market information and rational expectations, in. H Hauptmann, W Knelle and K. Moslers. eds. Operations research and economic theory (Springer, New York)