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PRIZES VERSUS WAGES WITH ENVY AND PRIDE*

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We show that if agents are risk-neutral, prizes outperform wages if and only if there is sufficient pride and envy relative to the noisiness of performance. If agents are risk-averse, prizes are a necessary supplement to wages (as bonuses).
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1. Introduction

Prizes are the simplest among contracts that reward agents based on their relative performance: agents' outputs are ranked, and the highest output is given a predetermined prize. On the other hand, *wages* are purely individual contracts, paid to an agent based on his output alone and regardless of what others are doing. The motivating power of prizes versus wages has been most famously considered in Lazear and Rosen (1981), who showed that both contracts are efficient as long as agents are risk-neutral. In a follow-up paper, Green and Stokey (1983) argued that, if agents are risk-averse and if their productivities are sufficiently correlated via a common random shock, then prizes outperform wages from the principal's point-of-view. The reason is that the incentives for agents to work, generated by wages, are reduced on account of the shock and the risk-aversion; while the incentives generated by prizes are invariant of the shock because it is common.

But, without correlation, can prizes still outperform wages?

It turns out that they can, provided agents have "other-regarding" preferences over their rewards. In a pioneering paper, Itoh (2004) characterized optimal contracts in a binary framework with two possible effort levels of the agents (work and shirk) and two possible output levels (success and failure) that are independent across agents. In particular Itoh showed that a prize contract is optimal if agents care about their *status* vis-à-vis others, feeling *envy* (loss in utility) when their reward is lower, and *pride* (gain in utility) when it is higher.[1] Such concern for status seems to be prevalent in practice. Indeed there is a large empirical literature, starting from Easterlin (1974), who argued that happiness depends not just on absolute, but also on relative, consumption.[2]

* It is a pleasure to thank two anonymous referees for useful suggestions, and also for making us more aware of the relevant literature.

[1] See Proposition 4, case (2a), in Itoh (2004). Note also that Itoh does not use the words "envy" and "pride" — this terminology is ours.

[2] This externality, stemming from status concerns, has been formally modelled along two different lines. The cardinal approach makes utility depend on the *difference* between an individual's consumption and others' consumption (see, e.g. Duesenberry (1949), Pollak (1976)). The ordinal approach makes utility depend on the *individual's rank* in the distribution of consumption (see, e.g. Frank (1985), Direr (2001), and Hopkins and Kornienko (2004)). The model of Itoh (2004) and the one presented in this paper are in the cardinal tradition. The ordinal approach is examined in Dubey and Geanakoplos (2005).

Itoh's (2004) specification of agents' utilities follows the simple functional form proposed in Fehr and Schmidt (1999), with one important difference. Fehr and Schmidt (1999) postulate "inequity aversion", i.e. any deviation of an agent's reward from another's results in a loss of utility: he feels "compassion" if he is ahead of his rival, and envy if he is behind, losing utility in either case. In Itoh's (2004) framework, compassion is permitted, but not required. Itoh considers envy in conjunction with either pride or compassion. With envy and pride (henceforth, E&P), prizes outperform wages in Itoh's model; when compassion replaces pride, prizes can still be effective, but only when they take the form of a "team prize", i.e. a prize which is shared equally by everyone if, and only if, all of them achieve success simultaneously (see Itoh (2004) for details). Our focus here is on the standard prize, and on the delineation of regimes when such prizes outperform wages or vice versa, once we step outside the world of binary outputs and allow for noise.

Itoh's result is very sharp: the optimal contract consists of a prize[3] provided there is *any* positive degree of E&P, no matter how small. However, this does not seem to be consistent with what is observed in labor markets: even though arguably most people are not immune to E&P, it is not often that they work *purely* for prizes.

Our analysis shows that Itoh's conclusion needs to be modified if his binary framework is replaced by one with a continuum of output levels (such a continuum is better suited for many applications, and also permits the modeling of random unbiased noise in output, which is independent of effort). First suppose there is no E&P. When there is also no noise, wages induce the same incentives as prizes for risk-neutral agents, and outperform prizes for risk-averse agents. However, with the introduction of noise, wages strictly dominate prizes not just with risk-aversion, but even risk-neutrality: since outputs are independent, compensating a worker on the basis of relative performance only distorts his incentives (the shirker wins the prize with positive probability just because of luck).

By a continuity argument, wages also dominate prizes for *small* E&P. In fact this is so until E&P becomes "sufficiently big". The precise analysis is carried out in Section 2, which examines the case of two risk-neutral agents. We show that for any level of noise below a certain bound, there is a threshold of E&P such that prizes outperform wages for E&P above the threshold, but wages outperform prizes for E&P below the threshold. Furthermore there is a second, larger bound such that when the noise level exceeds it, no amount of E&P can restore the superiority of prizes. Thus Itoh's conclusion regarding the superiority of prizes in the case of risk-neutral agents remains valid, but under two conditions: the level of random noise must be sufficiently low, and E&P must be sufficiently high. In the regime where noise is high, wages are always better than prizes.

Now consider risk-averse agents. Even when there is no noise in output, agents with the same skill that work for a prize and exert identical effort still face a 50% uncertainty about who will get it, which is not the case with wages. Thus risk-aversion will cause wages to outperform prizes; however, Itoh's (2004) intuition, that relative performance should not be ignored, still holds. We present robust conditions in Section 3 under which wages, *supplemented* by prizes (bonuses), constitute an improvement on wages alone. Bonuses are quite common in practice in labor markets.

In the basic verson of our model, we confine ourselves to just two agents, and assume a linear (piece-rate) wage structure. This assumption only strengthens our conclusions on

[3] The prize in the optimal contract is given only in the case of clear victory: its recipient must succeed and all his rivals must fail.

the superiority of wages. Even without this assumption, we find in Section 4 that, in many instances, prizes outperform non-linear wages when noise is small. When there are more than two agents, the main message of Itoh is further reinforced. We show in Section 5 that, no matter how large the noise and how small the E&P, the superiority of prizes is restored when the group of competing agents is big enough, since a shirker will very rarely be lucky enough to pass so many hard-working rivals.

Our work, as was said, is most directly linked to Itoh (2004). But several related papers must be mentioned which also focus on incentives generated by "other-regarding" preferences. As we note in Section 2.3, status-seeking agents are easier to motivate than status-neutral agents. This was shown in Grund and Sliwka (2005) in the context of tournaments (prizes), and in Neilson and Stowe (2010) for wages. However, neither compared wages to prizes in terms of their efficacy in eliciting effort. In a variation of Itoh's theme, Rey-Biel (2008) considered (either status-seeking or inequity-averse) agents who can observe not only their rivals' rewards but also the underlying effort levels, and thus measure their distances from each other in terms of net utilities. He showed that, when there is no uncertainty in production,[4] globally optimal contracts must inflict extreme inequality on any agent who unilaterally chooses to shirk, thus lending support to Itoh's (2004) extreme prize contract. In general, however, optimal labor contracts have been examined in the context of inequity-averse, rather than status-seeking, agents, with markedly different conclusions. For instance, Englmaier and Wambach (2010) (who, as Dur and Glazer, (2008), also assume that the agent may compare his compensation to the principal's share of the profit) show that team incentives[5] are in general necessary,[6] and purely competitive contracts, such as prizes, fare badly. Also, in the special case of inequity aversion when agents feel only envy, Bartling and von Siemens (2010) showed that the envy can in fact have negative implications for the principal's profit if unlimited liability (negative rewards) can be imposed on the agents — indeed, if envy is reduced, the principal can implement the same effort level with less payout.

2. Envy and pride

2.1 The basic model

We consider two identical agents with utility

$$u(A, B, e) = A + \beta \max(A - B, 0) - \alpha \max(B - A, 0) - ce,$$

where A is the money the agent gets, B is the money his rival gets, and e is the effort he exerts.[7] The parameters $\beta \geq 0$ and $\alpha \geq 0$ correspond to pride and envy, and $c > 0$ is the marginal disutility of effort.

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4 In our case, as in much of the literature, only outputs are observable; and since there is a random noise component to them, the effort from which they arose cannot be inferred. Each agent only needs to know the distribution of *outputs* (and thereby that of the piece-rate or prize rewards) of his rivals in order to calculate his payoff in our game.

5 That is, contracts where the reward of an agent depends positively on the output of the other agents.

6 Bartling (2011) argued that, even in the presence of correlation in production and risk-averse agents, pure team contracts may nonetheless be optimal as they are inequality-reducing.

7 Utility functions of this form were considered, e.g. in Kirchsteiger (1994), Bolle (2000), Fershtman *et al*. (2003), and Itoh (2004). Fehr and Schmidt (1999) considered this particular utility function but took β

Let a finite $\mathcal{E} \subset [0, 1]$ be the set of effort levels available to each agent, with $0 \in \mathcal{E}$ and $1 \in \mathcal{E}$ (thus, we require that it contains two special levels: 0 ≡ "shirking", and 1 ≡ "working at full capacity"). If agent $i \in \{1, 2\}$ chooses effort level $e_i \in \mathcal{E}$, he produces $e_i + \varepsilon_i^\sigma$ units of output, where ε_1^σ and ε_2^σ are random noises (i.i.d. non-atomic random variables with mean zero), parameterized by a scalar $\sigma > 0$ measuring their noisiness.[8] We denote by G^σ the cumulative distribution function of the random variable $\varepsilon_1^\sigma - \varepsilon_2^\sigma$. Clearly, since ε_1^σ and ε_2^σ have positive variance and are non-atomic i.i.d. random variables, we have $G^\sigma(0) = 1/2$. We suppose that as noise disappears, $\lim_{\sigma\to 0} G^\sigma(t) = 0$ for every $t < 0$, and as noise goes to infinity, $\lim_{\sigma\to\infty} G^\sigma(t) = 1/2$ for every t. We also assume that G^σ is continuous and convex on $[-1, 0]$ (i.e. G^σ possesses a density function which is non-decreasing on $[-1, 0]$).

To include deterministic output in our analysis, we also allow for $\sigma = 0$, in which case both ε_1^0 and ε_2^0 are fixed at zero.

If each ε_i^σ is *normally distributed*, with mean zero and standard deviation σ, then $\varepsilon_1^\sigma - \varepsilon_2^\sigma$ is also normally distributed, with mean zero and standard deviation $\sqrt{2}\sigma$; thus, $G^\sigma(x) = \frac{1}{2\sigma\sqrt{\pi}} \int_{-\infty}^{x} \exp\left(-\frac{t^2}{4\sigma^2}\right) dt.$

If the ε_i^σ are *uniformly distributed* on $[-\sigma, \sigma]$, then

$$G^\sigma(x) = \begin{cases} 0, & \text{if } x \le -2\sigma, \\ \frac{1}{8\sigma^2}(x + 2\sigma)^2, & \text{if } -2\sigma \le x \le 0, \\ 1 - \frac{1}{8\sigma^2}(-x + 2\sigma)^2, & \text{if } 0 \le x \le 2\sigma, \\ 1, & \text{if } x \ge 2\sigma. \end{cases}$$

It is easy to check that all our hypotheses are satisfied for the normal and uniform noise terms.

2.2 The wage and prize games

We will compare two types of contracts that the principal may write. The first is a piece-rate wage contract: each agent is paid rq, when the piece-rate is r and his output is q. In the second contract, a prize P is awarded to the agent with the highest output; in case of ties, a fair coin is tossed to decide who gets the prize. There is always one winner.

Each of these contracts induces, in an obvious manner, a non-cooperative game in which agents' strategies are to choose effort levels. Denote these games with wages, prizes by $\Gamma^\sigma_{\alpha,\beta}(r)$, $\tilde{\Gamma}^\sigma_{\alpha,\beta}(P)$.

The principal wishes to elicit maximal effort from the agents (i.e. $e_1 = e_2 = 1$) at minimal expected cost to himself. Let

negative, implying that people feel compassion when they are ahead. In conjunction with envy from being behind, their formulation amounts to "inequity aversion".

[8] Our model allows for negative outputs of the agents. This might make sense in certain contexts (think of money managers who make losses). But the case of exclusively non-negative outputs can be incorporated by putting a positive lower bound on effort levels and a suitably small upper bound on the support of the random noise.

$$M^{\sigma}_{\alpha,\beta} = 2\min\{r \mid (e_1 = 1, e_2 = 1) \text{ is a Nash equilibrium of } \Gamma^{\sigma}_{\alpha,\beta}(r)\},$$

$$\tilde{M}^{\sigma}_{\alpha,\beta} = \min\{P \mid (e_1 = 1, e_2 = 1) \text{ is a Nash equilibrium of } \tilde{\Gamma}^{\sigma}_{\alpha,\beta}(P)\}.$$

Clearly $M^{\sigma}_{\alpha,\beta}$, $\tilde{M}^{\sigma}_{\alpha,\beta}$ is the minimal expected payment by the principal needed to elicit maximal effort via wages, prizes.[9]

Our first proposition establishes explicit formulae for $M^{\sigma}_{\alpha,\beta}$ and $\tilde{M}^{\sigma}_{\alpha,\beta}$.

Proposition 1: *Let*

$$\psi^{e}_{\sigma} \equiv E\left[\max\{e + \varepsilon^{\sigma}_1 - 1 - \varepsilon^{\sigma}_2, 0\}\right]$$

for every $e \in \mathcal{E}\backslash\{1\}$*, and let*

$$\Delta^{e}_{\sigma} \equiv \psi^{1}_{\sigma} - \psi^{e}_{\sigma}. \tag{2.1}$$

(It is easy to see that $0 \le \frac{\Delta^{e}_{\sigma}}{1-e} \le \frac{1}{2}$*). Denote*

$$\Delta_{\sigma} \equiv \begin{cases} \max_{e\in\mathcal{E}\backslash\{1\}} \dfrac{\Delta^{e}_{\sigma}}{1-e}, & \text{if } \beta \le \alpha, \\ \min_{e\in\mathcal{E}\backslash\{1\}} \dfrac{\Delta^{e}_{\sigma}}{1-e}, & \text{if } \beta > \alpha. \end{cases} \tag{2.2}$$

Then

$$M^{\sigma}_{\alpha,\beta} = \frac{2c}{1 + \alpha + (\beta - \alpha)\Delta_{\sigma}}, \tag{2.3}$$

and

$$\tilde{M}^{\sigma}_{\alpha,\beta} = \frac{c}{\frac{1}{2} - G_{\sigma}(-1)} \cdot \frac{1}{1 + \alpha + \beta}. \tag{2.4}$$

Proof: See the Appendix. ■

When there is no envy or pride, i.e. when $\alpha = \beta = 0$, then by Proposition 1 $M^{\sigma}_{0,0} = 2c$ and $\tilde{M}^{\sigma}_{0,0} = \frac{c}{\frac{1}{2} - G_{\sigma}(-1)}$. The following result is therefore an obvious corollary:

Theorem 1: *If there is no envy or pride, then wages are never worse than prizes:* $M^{\sigma}_{0,0} \le \tilde{M}^{\sigma}_{0,0}$ *for any* σ. *Furthermore if there is sufficient noise (*$G^{\sigma}(-1) > 0$*), then wages outperform prizes:* $M^{\sigma}_{0,0} < \tilde{M}^{\sigma}_{0,0}$.

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[9] We have assumed a *single* prize for the best-performing agent. If the loser were also awarded, incentives to exert maximal effort would become smaller when $\alpha = \beta$ (otherwise, with an appropriate upper bound on α). Thus a single prize will, in fact, be preferred by the principal.

The intuition behind Theorem 1 is straightforward. Suppose $\mathcal{E} = \{0, 1\}$. If agent i works ($e_i = 1$) in the prize game and so does his rival, i's expected share of the prize is exactly $P/2$. If he shirks ($e_i = 0$) and his rival still works, his expected payoff does not fall to zero, since with noise he may, with a stroke of luck, win anyway. His expected payoff is $G^{\sigma}(-1)P$. On net his incentive to work (i.e., the increase in agent's payoff when he switches from shirk to work, ignoring his disutility of effort and assuming that his rival is working) is $P(1/2 - G^{\sigma}(-1))$. When the wage rate is set equal to $P/2$, his incentive to work in the wage game is $P/2$, no matter what the noise. But if $G^{\sigma}(-1) > 0$, then $P(1/2 - G^{\sigma}(-1)) < P/2$. Hence the prize P will need to be more than twice the optimal wage r if $G^{\sigma}(-1) > 0$, and will never be less.

2.3 The power of envy and pride

E&P make it easier to motivate the agents to work, via wages or prizes.[10] For wages, this is because shirking entails not only a lesser payment, but also the envy of those who are working and getting paid more.

But the motivating power of envy and pride is even stronger with prizes than with wages. Notice that an agent who shirks not only reduces his (expected) prize, he *increases* the (expected) prize of his rival, generating still more envy.[11] Indeed, Itoh (2004) established in his binary framework (where there are *only* two output levels — success or failure) that, whenever envy and pride are present (no matter to how small a degree), an extreme type of contract is optimal: a prize should be given to the agent who succeeds when his rival fails, and no prize should be given in any other circumstance.

The efficacy of a prize can be clearly seen in our model when there is no noise ($\sigma = 0$). Let us assume (as in Itoh (2004) and Fehr and Schmidt (1999)) that the ratio between α and β is constant:

$$\beta = \gamma \cdot \alpha \tag{2.5}$$

for some fixed $\gamma > 0$. Since the envy parameter α now also determines the pride parameter β, we shall call α the *envy-pride (E-P)* parameter. From (2.3) and (2.4) we see at once that, when $\sigma = 0$, the principal needs to pay out total wages $M^0_{\alpha,\gamma\cdot\alpha} = 2c/(1+\alpha)$, but a prize of only $\tilde{M}^0_{\alpha,\gamma\cdot\alpha} = 2c/(1+\alpha+\gamma\alpha)$, in order to motivate both agents to work. Clearly both the required wage bill and the prize become smaller as the E-P parameter α rises. When $\alpha = 0$, $M^0_{0,0} = \tilde{M}^0_{0,0} = 2c$ whereas both $M^0_{\alpha,\gamma\cdot\alpha}$ and $\tilde{M}^0_{\alpha,\gamma\cdot\alpha}$ converge to zero as $\alpha \to \infty$. For high enough α, the principal hardly needs to expend any money at all. But the point is, he expends *less* on prizes than on wages, i.e. $M^0_{\alpha,\gamma\cdot\alpha} > \tilde{M}^0_{\alpha,\gamma\cdot\alpha}$, for *any* $\alpha > 0$.

The presence of noise, however, changes the situation in a crucial way, which Itoh's (2004) binary framework cannot account for. As was seen in Theorem 1, if there is no E-P, then a modicum of unbiased noise in output leaves wages intact but harms prizes: if $G^{\sigma}(-1) > 0$, wages outperform prizes ($M^{\sigma}_{0,0} < \tilde{M}^{\sigma}_{0,0}$). By an obvious continuity argument this will also be the case for small but positive E-P ($M^{\sigma}_{\alpha,\gamma\cdot\alpha} < \tilde{M}^{\sigma}_{\alpha,\gamma\cdot\alpha}$ for sufficiently small $\alpha > 0$). Thus Itoh's conclusion that prizes are superior to wages for any $\alpha > 0$ is not true

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10 As E&P increase, agents are obviously more easily motivated to work (see Proposition 1). This fact was noted in Grund and Sliwka (2005) in the context of tournaments, and in Neilson and Stowe (2010) for wages.

11 Notice that this effect relies on the cardinal approach to E&P: E-P increase as the gap grows bigger.

in the context of a continuum of outputs. Our next theorem shows that, if the noise in output is below a certain bound, a *sufficiently high* E-P is *necessary and sufficient* for prizes to outperform wages.

Theorem 2: *Suppose that noise is not too large:*

$$G^{\sigma}(-1) < \min\left(\frac{1}{4}, \frac{\gamma}{2(1+\gamma)}\right). \tag{2.6}$$

Define the noise-dependent threshold

$$\alpha^* = \frac{2G^{\sigma}(-1)}{\gamma - 2(1+\gamma)G^{\sigma}(-1) + (1-\gamma)\Delta_{\sigma}}. \tag{2.7}$$

If E-P exceeds the threshold, then prizes outperform wages: $\tilde{M}^{\sigma}_{\alpha,\gamma\cdot\alpha} < M^{\sigma}_{\alpha,\gamma\cdot\alpha}$ *if* $\alpha > \alpha^*$. *If E-P is below the threshold, then wages outperform prizes:* $M^{\sigma}_{\alpha,\gamma\cdot\alpha} < \tilde{M}^{\sigma}_{\alpha,\gamma\cdot\alpha}$ *if* $\alpha < \alpha^*$.

Proof: See the Appendix. ■

The following special case brings out the intuition behind Theorem 2. Suppose ε_1^{σ} and ε_2^{σ} are normally distributed with mean zero and standard deviation σ, and also suppose $\gamma = 1$. Condition (2.6) on the noise is thus satisfied whenever $\sigma \leq 1$, since then

$$G^{\sigma}(-1) \leq G^{1}(-1) \approx 0.24 < \frac{1}{4}.$$

By (2.3) and (2.4),

$$M^{\sigma}_{\alpha,\alpha} = \frac{2c}{1+\alpha} \quad \text{and} \quad \tilde{M}^{\sigma}_{\alpha,\alpha} = \frac{c}{\frac{1}{2} - G_{\sigma}(-1)} \cdot \frac{1}{1+2\alpha}. \tag{2.8}$$

As was already said, when there is no noise, i.e. $\sigma = 0$, wages are strictly worse than prizes ($M^{0}_{\alpha,\alpha} > \tilde{M}^{0}_{\alpha,\alpha}$) except when there is no E-P ($\alpha = 0$). However (2.8) shows that when noise is positive, i.e. $0 < \sigma \leq 1$, the expenditure on wages remains the same ($M^{\sigma}_{\alpha,\alpha} = M^{0}_{\alpha,\alpha}$), but the expenditure on a prize contract increases ($\tilde{M}^{\sigma}_{\alpha,\alpha} > \tilde{M}^{0}_{\alpha,\alpha}$). Thus noise worsens the case for prizes over wages, but the comparison of the terms $M^{\sigma}_{\alpha,\alpha}$ and $\tilde{M}^{\sigma}_{\alpha,\alpha}$ in (2.8) immediately shows that prizes retain their superiority over wages when α is sufficiently big, i.e, above the positive threshold $\alpha^* = \frac{2G^{\sigma}(-1)}{1-4G^{\sigma}(-1)}$; while wages do better below the threshold (to get a feel for the magnitude, notice for instance that, if $\sigma = 1/2$, then $\alpha^* \approx 0.23$, i.e. agents need to care one-quarter as much about the gap in payments as about their own payment in order for prizes to dominate wages[12]).

Theorem 2 is in line with the results obtained by Itoh (2004) in the *binary* framework: if, as in the binary output case, there is no continuous noise in output (or very little of it, implying $G^{\sigma}(-1) = G^{0}(-1) = 0$), then $\alpha^* = 0$, and hence prizes outperform wages with the slightest E-P. More generally, our third Theorem states that given *any* positive E-P α (however small), prizes outperform wages provided the random noise in agents' outputs is sufficiently low (below some upper bound that depends on α).

Theorem 3: *Given* $\alpha > 0$, *there exists* $\sigma' > 0$ *such that whenever* $\sigma \leq \sigma'$, $\tilde{M}^{\sigma}_{\alpha,\gamma\cdot\alpha} < M^{\sigma}_{\alpha,\gamma\cdot\alpha}$.

Proof: See the Appendix. ■

Our next result emphasizes one drawback of prizes, which is not revealed in the framework of Itoh (2004): too much noise destroys their efficacy, no matter how much E-P there may be. The reason is as follows. When wages are based on a noisy measure of output, a worker may be overpaid or underpaid w.r.t. his effort. But as long as the noise is unbiased, and wages are linear, his expected wage is correct. In contrast, when prizes are based on a noisy measure of relative output, the expected payment a worker gets is biased toward $P/2$, diminishing the expected payment to the hard worker and increasing the expected payment to the shirker.

Theorem 4: *Suppose that noise is sufficiently large:*

$$G^{\sigma}(-1) \geq \max\left(\frac{1}{4}, \frac{\gamma}{2(1+\gamma)}\right). \tag{2.9}$$

Then wages outperform prizes no matter what the E-P is: $M^{\sigma}_{\alpha,\gamma\cdot\alpha} < \tilde{M}^{\sigma}_{\alpha,\gamma\cdot\alpha}$ *for every* $\alpha \geq 0$.

Proof: See the Appendix. ■

Remark 1 (pride vs envy): Assumption (2.5), which rigidly ties the envy and pride parameters through the ratio γ, is conducive to the neat statement of Theorem 2, in terms of a *single* threshold value for E-P. But it precludes an inquiry into the separate roles that pride and envy may play in the efficacy of the prize. Now let us drop asumption (2.5) and consider the simple scenario with no noise ($\sigma = 0$) and binary effort levels $\mathcal{E} = \{0, 1\}$. The incentive to work for wage rate r is

$$r + \alpha r.$$

This is the sum of the direct utility of consuming the wage r, and the envy αr suffered when shirking and getting zero while the rival works and gets r. The payoff to an agent who works in the prize game, excluding disutility from work, is $\frac{1}{2}(P + \beta P) + \frac{1}{2}(-\alpha P)$; if he shirks, he gets $-\alpha P$. Thus the incentive to work with prize P is thus

[12] Note that, with normally distributed noise, the principal *collects* money from an agent with positive probability in a wage contract (whenever the agent produces negative output, i.e. a "loss"). With prizes, he only *hands out* money. In spite of this, the principal prefers prizes to wages when the level of E-P is sufficiently high.

$$\frac{1}{2}P + \frac{1}{2}\alpha P + \frac{1}{2}\beta P.$$

Setting the prize fund P equal to the total wage bill $2r$, we see that prizes provide an extra incentive of βr. Thus no matter how large or small envy α is, the slightest presence of pride ($\beta > 0$) will cause prizes to outperform wages; and indeed, formally, using Proposition 1 for $\beta > 0$ and $\sigma = 0$ we obtain

$$M^{0}_{\alpha,\beta} = \frac{2c}{1+\alpha} > \frac{c}{\frac{1}{2}} \cdot \frac{1}{1+\alpha+\beta} = \tilde{M}^{\sigma}_{\alpha,\beta}. \tag{2.10}$$

With noise in output ($\sigma > 0$), a simple continuity argument extends the above claim, showing that prizes outperform wages provided the noise σ is sufficiently small; and a more delicate analysis can show that, with *any* fixed (but not too high) level of noise, sufficiently high pride has the effect of making prizes superior to wages (provided the envy parameter does not exceed some fixed multiple of the pride parameter). For the details, see Section 2.4 of the working paper Dubey *et al*. (2011) of which the current article is a subset.

3. When agents are not risk-neutral: The need for a bonus

The biggest objection to prizes is that they force agents to face a huge uncertainty about who will get the prize, even if they work hard. It comes to the fore when agents are risk-averse. Indeed we will see in Remark 2 that, even when there is no noise in output, risk-aversion causes wages to outperform prizes. But Itoh's (2004) intuition regarding prizes still holds in some measure. We find (see Theorem 5) that, with positive envy and pride, wages supplemented by prizes are in many instances an improvement on wages alone.[13] These supplementary prizes are common in practice, in the form of bonuses.

Let the utility function of each of the two agents be given by

$$u(A, B, e) = U(A) - \alpha V(\max\{B - A, 0\}) + \beta V(\max\{A - B, 0\}) - ce,$$

where, as before, A is the amount paid to the agent, B the amount paid to his rival, α his envy parameter, β his pride parameter, e his choice of effort level, and $c > 0$ the disutility from effort.[14] We assume that U and V are continuously differentiable, and that their derivatives are strictly positive everywhere[15] (since agents like rewards; and since envy/

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[13] In the context of risk- and inequity-averse agents, it has already been pointed out by Englmaier and Wambach (2010) that an independent contract is in general not optimal, and that a globally optimal contract should include team incentives (i.e. the reward of an agent should depend positively on the output of the other agent). Here, with status-seeking agents for whom $\beta > 0$, we show the opposite, that prize supplements (under which the individual reward depends negatively on the output of the other agent) are often needed.

[14] We could have more generally considered $u(A, B, e) = U(A) - \alpha V_{envy}(\max\{B - A, 0\}) + \beta V_{pride}(\max\{A - B, 0\}) - ce$ instead of supposing $V = V_{pride} = V_{envy}$. Similar results would obtain but at the cost of more notation.

[15] U is defined on $\mathbb{R}$, while V on $\mathbb{R}_{+}$.

pride must increase the more one is behind/ahead of one's rival). Furthermore, both U and V vanish at zero (since there is no envy/pride in being ahead/behind). We do *not* assume that either U or V is concave.

For simplicity, we take the i.i.d. random noises ε_1^σ and ε_2^σ to be bounded, with support on a compact interval $[-\lambda, \lambda]$, for all σ. As before, G^σ denotes the cumulative distribution function of $\varepsilon_1^\sigma - \varepsilon_2^\sigma$, and we assume that G^σ is convex on $[-1, 0]$.

Up until now we only considered "pure" contracts which could take the form of either a prize P or a piece-rate wage r. Now we allow for *mixed* contracts (P, r): each agent is paid rq when his output is q, plus a prize (bonus) P if his output is more than his rival's (tossing a coin in case of ties). The contract (P, r) induces[16] a game $\Gamma_{\alpha,\beta}^\sigma(P,r)$ in the obvious manner.

Let $\Pi_{\alpha,\beta}^\sigma$ denote the set of mixed contracts which elicit full effort, i.e.

$$\Pi_{\alpha,\beta}^\sigma = \{(P,r) \in R_+^2 \mid (e_1 = 1, e_2 = 1) \text{ is a Nash equilibrium of } \Gamma_{\alpha,\beta}^\sigma(P,r)\}.$$

The principal's payout is $P + 2r$ when $(e_1 = 1, e_2 = 1)$ is played in $\Gamma_{\alpha,\beta}^\sigma(P,r)$. Thus the set of *optimal* contracts is

$$\tilde{\Pi}_{\alpha,\beta}^\sigma = \operatorname{argmin}\{P + 2r \mid (P,r) \in \Pi_{\alpha,\beta}^\sigma\}.$$

With risk-neutral agents, there is no need to consider $\Pi_{\alpha,\beta}^\sigma$ because pure contracts are just as good as any mixture: there always exists $(P,r) \in \tilde{\Pi}_{\alpha,\beta}^\sigma$ such that either $P = 0$ or $r = 0$, at least in the canonical case of $\mathcal{E} = \{0, 1\}$, since then the (unique) incentive constraint defining $\Pi_{\alpha,\beta}^\sigma$ is linear in r and P.

If agents are *not* risk-neutral, however, mixed contracts may well beat pure contracts. We leave the exploration of the exact structure of optimal mixed contracts for future research. But we shall delineate two scenarios in which *any optimal mixed contract must necessarily entail a positive bonus*, i.e. $P > 0$ for every $(P,r) \in \tilde{\Pi}_{\alpha,\beta}^\sigma$.

In the first scenario (Theorem 5 below) envy and pride are fixed at an arbitrary positive level. It turns out that bonuses are needed, provided the noise is sufficiently small. Moreover the optimal contract may often not be a pure prize since, with risk-aversion, pure prize tends to be inferior to pure wage (see Remark 2 below). In this case, both the wage and the prize (bonus) components of the optimal contract (P, r) will be positive. In the second scenario (Theorem 6 below), the noise is fixed and not too large. Here for sufficiently high envy and pride, even pure prizes will beat wages, again showing the general need for bonuses.

The intuition for Theorem 5 is roughly as follows. Suppose the two agents are earning only wages. When there is no noise, a hard-working agent knows the wage $w = r \cdot 1$ he will earn for sure. Assuming differentiable utilities, he is nearly risk-neutral for small variations in consumption. So consider reducing the piece-rate by ε, and instead awarding a prize of 2ε to the highest performance. Then the expected consumption utility of a hard-working agent stays almost the same. But as we argued before with risk-neutrality, the incentive created by envy-pride is greater for the prize than the wage.

[16] The underlying components c, U, V of the utility are held fixed, while (P, r), σ, α, and β vary.

Thus a small bonus increases incentives without increasing the total expected payout of the principal.[17]

Note that this argument only works for small prizes and small noise. As the prize gets larger, risk-aversion kicks in and the prize becomes a less attractive substitute for wages. As noise increases, the luckiest worker, who already has the highest wage and therefore the lowest marginal utility for money, will get the prize, reducing its ex ante consumption utility.

Theorem 5: *Bonus is needed with sufficiently low noise, for fixed E&P. To be precise, assume that: (i)*[18] $\alpha \leq \beta$*; and (ii) there exist* $B < \infty$ *and* $b > 0$ *such that* $U'(x) \leq B$ *for every* $x \in \mathbb{R}$ *and* $b \leq V'(x) \leq B$ *for every* $x \in \mathbb{R}_+$*. Then there exists* $\sigma' > 0$ *such that* $P > 0$ *for every* $(P, r) \in \tilde{\Pi}^{\sigma}_{\alpha,\beta}$ *whenever* $\sigma \leq \sigma'$.

Proof: See the Appendix. ■

Remark 2 (optimal contracts tend to be strictly mixed): Assume that there are just two effort levels, i.e. $\mathcal{E} = \{0, 1\}$ and that the agents are strictly risk-averse, i.e., U is strictly concave. If there is no noise ($\sigma = 0$) and no envy-pride ($\alpha = \beta = 0$), it is obvious that the minimal prize that implements ($e_1 = 1$, $e_2 = 1$) as a Nash equilibrium in the prize game $\tilde{\Gamma}^0_{0,0}$ is $\tilde{M}^0_{0,0} = U^{-1}(2c)$ and the minimal wage payout by the principal that implements ($e_1 = 1$, $e_2 = 1$) as a Nash equilibrium in the wage game $\Gamma^0_{0,0}$ is $M^0_{0,0} = 2U^{-1}(c) < U^{-1}(2c)$. Consequently, for all sufficiently low E&P and noise, $M^{\sigma}_{\alpha,\beta} < \tilde{M}^{\sigma}_{\alpha,\beta}$ This shows that the set of optimal contracts $\tilde{\Pi}^{\sigma}_{\alpha,\beta}$ does *not* contain a pure prize contract. Since it does not contain a pure wage contract either (by Theorem 5), we conclude that any optimal contract is strictly mixed: $P > 0$ and $r > 0$ for every $(P, r) \in \tilde{\Pi}^{\sigma}_{\alpha,\beta}$.

Corollary 1 below supplements Theorem 5 and shows that for any fixed noise (below some reasonably large upper bound) a bonus is again needed if there is enough envy and pride. The intuition for this result is that as envy and pride get very large, the optimal piece-rate (assuming no prize) goes to zero. Since the noise is bounded, the final consumption, being the product of the piece-rate and output, also goes to zero. Thus consumption is practically certain, and the agents become nearly risk-neutral. Hence, as in the previous sections, even the pure prize outperforms wages:

Theorem 6: *Prizes outperform wages even without risk-neutrality, provided there is sufficient E-P and noise is not too large. To be precise, suppose that* $\beta = \gamma \cdot \alpha$ *for a fixed* $\gamma > 0$ *and that condition (2.6) of Theorem 2 holds. Then there exists* $\alpha' > 0$ *such that* $\tilde{M}^{\sigma}_{\alpha,\gamma\cdot\alpha} < M^{\sigma}_{\alpha,\gamma\cdot\alpha}$ *if* $\alpha > \alpha'$.

Proof: See the Appendix. ■

The following is an obvious corollary of Theorem 6:

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[17] Note that this intuition is somewhat deficient, as a major difference remains between the risk-neutral and the non-risk-neutral cases even when the added prize component is infinitesimal. The *marginal* envy, or pride, may change when a non-risk-neutral agent switches from work to shirk, distorting in a certain way the incentive to work compared to the exact risk-neutral case. This will be seen in the proof of Theorem 5; it is for this reason that the assumption that $\alpha \leq \beta$ is made in this theorem.

[18] Assumption (i) can be substituted by requiring that V be *convex*.

Corollary 1: *A bonus is needed with sufficiently high E-P, given any fixed and not too large noise: under the assumptions of Theorem 6, there exists* $\alpha' > 0$ *such that, if* $\alpha > \alpha'$, *then* $P > 0$ *for every* $(P, r) \in \tilde{\Pi}^{\sigma}_{\alpha,\gamma\cdot\alpha}$.

4. Non-linear wages

If we were to allow non-piece-rate contracts, based on more general wage functions, the performance of wage contracts would obviously improve. Thus wages would outperform prizes for sufficiently small levels of envy-pride or sufficiently big noise in output, as implied by Theorems 2 and 4. However we will show here that prizes may outperform even non-linear wages when noise is *small*.

A non-linear wage is given by a function w, defined for all possible outputs. These functions are assumed to be non-decreasing, bounded from above by some constant $W > 0$, and to have the property that expected wages are nonnegative even with zero effort, i.e. $Ew(\varepsilon_i^{\sigma}) \geq 0$ for $i = 1, 2$. This guarantees that agents do not get expected negative wages under any level of effort. Denote by $\bar{M}^{\sigma}_{\alpha,\beta}(w)$ the expected payment by the principal under wage function w, when both agents make effort 1. Also let $\bar{M}^{\sigma}_{\alpha,\beta}$ be the infimum of $\bar{M}^{\sigma}_{\alpha,\beta}(w)$ over all (non-linear) w which implement maximal effort by both agents in Nash equilibrium.

First suppose that there is no random noise at all ($\sigma = 0$): agent i's output precisely equals his effort e_i. It is easy to see that there is an optimal w achieving $\bar{M}^{0}_{\alpha,\beta}$. This w pays zero for all output levels below 1 (i.e. $w(x) = 0$ for $x < 1$), and $w(1)$ is the minimal payoff under which no agent i prefers $e_i = 0$ to $e_i = 1$ given that his opponent j is choosing $e_j = 1$. As in the computation of $M^{\sigma}_{\alpha,\beta}$ in the proof of Proposition 1, $w(1) = c/(1 + \alpha)$ and so

$$\bar{M}^{0}_{\alpha,\beta} = \frac{2c}{1+\alpha} = M^{0}_{\alpha,\beta}.$$

It now follows from (2.10) that

$$\tilde{M}^{0}_{\alpha,\beta} < \bar{M}^{0}_{\alpha,\beta} \tag{4.1}$$

for all $\beta > 0$. Thus, when there is no noise, prizes outperform *all* wage contracts for any given positive level of envy and pride. When the noise is sufficiently low, this continues to hold, at least when $\alpha \geq \beta$:

Theorem 7: *Given* $\alpha \geq \beta > 0$, *there exists* $\sigma' > 0$ *such that* $\tilde{M}^{\sigma}_{\alpha,\beta} < \bar{M}^{\sigma}_{\alpha,\beta}$ *whenever* $\sigma \leq \sigma'$.

Proof: See the Appendix. ■

5. Multiple agents

When there are many agents, the scope for envy and pride increases. Coming first (or last) among 100 contestants *may* give more pleasure (or pain) than beating a single opponent. The principal could then take advantage of this fact to pay less, whether he uses wages or prizes. We suppress this possibility, and assume that agents care only about the *average* of others' receipts.

Multiple agents nevertheless have a beneficial effect on prizes. With two contestants, an agent who shirks might get lucky and beat the other agent who works. However, with 99 other agents working, the shirker is almost sure to come behind one of them. Thus sufficiently many agents tend to ameliorate the drawback of noise, helping prizes to become more efficacious than wages as long as there is some envy and pride.

Suppose that there are n identical agents. We assume that if agents $1, \ldots, n$ get $A_1, \ldots, A_n$ and agent i is exerting effort e_i, then i's utility is

$$u_i(A_1, \ldots, A_n, e_i) = A_i + \alpha\left(A_i - \frac{\sum_{j \neq i} A_j}{n-1}\right) - ce_i$$

(for simplicity, we take $\alpha = \beta$ and refer to their common value as the envy-pride (E-P) paramater). We also assume that the random noise variables $\left(\varepsilon_k^\sigma\right)_{k=1}^{\infty}$ have bounded support $[-\sigma, \sigma]$ and possess a continuously differentiable and strictly positive density function f^σ on it. It is shown in Lemmas 1 and 2 in the Appendix that, under these assumptions, there exists $N > 0$ such that the cumulative distribution function G_n^σ of the random variable $\left(\max_{1 \leq j \leq n, j \neq i} \varepsilon_j^\sigma\right) - \varepsilon_i^\sigma$ is convex on $[-1, 0]$ provided $n \geq N$, and

$$\lim_{n \to \infty} n^2 G_n^\sigma(-1) = 0. \tag{5.1}$$

As in Proposition 1, one can now see that

$$M_{\alpha,\alpha}^\sigma = \frac{nc}{1+\alpha}$$

and

$$\tilde{M}_{\alpha,\alpha}^\sigma = \frac{c}{1 + \dfrac{n}{n-1}\alpha} \cdot \frac{1}{\dfrac{1}{n} - G_n^\sigma(-1)}.$$

Furthermore, it is easy to verify the following analogue of Theorem 2:

Theorem 8: *Prizes outperform wages if, and only if, E-P exceeds a noise-dependent threshold. To be precise, suppose that*

$$n^2 G_n^\sigma(-1) < 1, \tag{5.2}$$

and define

$$\alpha^*(n) = \frac{n(n-1)G_n^\sigma(-1)}{1 - n^2 G_n^\sigma(-1)}. \tag{5.3}$$

If $\alpha > \alpha^*(n)$ *then* $\tilde{M}_{\alpha,\alpha}^\sigma < M_{\alpha,\alpha}^\sigma$, *and if* $\alpha < \alpha^*(n)$ *then* $\tilde{M}_{\alpha,\alpha}^\sigma > M_{\alpha,\alpha}^\sigma$.

Proof: Obvious amendments in the Proof of Theorem 2. ■

Due to (5.1), condition (5.2) holds for all large enough n. This means that the scenario described in Theorem 4 in the two-agent case is precluded when there is a sufficient number of agents: no matter how large the noise in output is, with a sufficient number of agents the E-P will kick in above some threshold, making prizes superior. Moreover, as will be spelled out in Theorem 9 below, the minimal level of E-P required for prizes to outperform wages becomes vanishingly small as the number of competitors increases; indeed, for any fixed levels of E-P and noise, prizes do better when there is a sufficient number of agents. In other words, multiple agents are a substitute for small noise as well as large E-P, the factors that drive the efficacy of prizes for two — or few — competitiors (see Theorems 2 and 3).

Theorem 9: *The threshold $\alpha^*(n)$ becomes vanishingly small as n increases: $\lim_{n\to\infty}\alpha^*(n) = 0$. In particular, given $\alpha > 0$ and $\sigma \geq 0$, there exists $n' > 0$ such that whenever $n \geq n'$, $\tilde{M}^{\sigma}_{\alpha,\alpha} < M^{\sigma}_{\alpha,\alpha}$.*

Proof: See the Appendix ■.

Theorem 9 may have an interesting implication for organizations which employ large peer groups in a common environment. If it is the case — and no doubt this needs empirical verification — that they are influenced by E-P, then prizes outperform wages once the peer group is large enough, no matter how small the intensity of the E-P may be and how big the noisiness in production (the group size required for this result will likely rise as the intensity falls or the noisiness increases.) It follows that in large peer groups, the principal should not offer a pure wage contract, but at least supplement it with a bonus. If agents happen to be risk-neutral as hypothesized in this section, then an optimal contract could well be a pure prize. But in the more realistic scenario that agents are risk-averse, our analysis of Section 3, taken in conjunction with Theorem 9, strongly suggests that the best policy for the principal would be to offer a mixed contract of wage plus bonus, once the peer group becomes large enough. And we reiterate that this will be so regardless of the levels of E-P or noise.

Appendix

Proof of Proposition 1:

In the game $\Gamma^{\sigma}_{\alpha,\beta}(r)$ the expected utility of agent i, when he chooses effort level e_i and his rival j chooses effort level e_j, is

$$re_i + \beta rE\left[\max\left\{e_i + \varepsilon^{\sigma}_i - e_j - \varepsilon^{\sigma}_j, 0\right\}\right] - \alpha rE\left[\max\left\{e_j + \varepsilon^{\sigma}_j - e_i - \varepsilon^{\sigma}_i, 0\right\}\right] - ce_i.$$

In order for ($e_1 = 1$, $e_2 = 1$) to be a Nash equilibrium of $\Gamma^{\sigma}_{\alpha,\beta}(r)$, it is necessary and sufficient that (under the piece-rate r) effort level 1 is not less attractive to an agent than any $e \in \mathcal{E}\backslash\{1\}$, given that his rival chooses effort level 1. Thus, we must have

$$r + \beta rE\left[\max\left\{1 + \varepsilon^{\sigma}_i - 1 - \varepsilon^{\sigma}_j, 0\right\}\right] - \alpha rE\left[\max\left\{1 + \varepsilon^{\sigma}_j - 1 - \varepsilon^{\sigma}_i, 0\right\}\right] - c$$

$$\geq re + \beta rE\left[\max\left\{e + \varepsilon^{\sigma}_i - 1 - \varepsilon^{\sigma}_j, 0\right\}\right] - \alpha r\left[\max\left\{1 + \varepsilon^{\sigma}_j - e - \varepsilon^{\sigma}_i, 0\right\}\right] - ce,$$

i.e.

$$r\left(1-\alpha\psi_\sigma^1+\beta\psi_\sigma^1\right)-c\geq r\left(e+\alpha\left(e-1-\psi_\sigma^e\right)+\beta\psi_\sigma^e\right)-ce,$$

for every $e \in \mathcal{E}\backslash\{1\}$ (here we use the obvious fact that $-E\left[\max\left\{1+\varepsilon_j^\sigma-e-\varepsilon_i^\sigma,0\right\}\right]+\psi_\sigma^e=e-1$). Thus, in order to implement (e_1= 1, e_2 = 1) as a Nash equilibrium of $\Gamma_{\alpha,\beta}^\sigma(r)$, it is necessary and sufficient that r satisfy

$$r\geq\frac{c(1-e)}{1-e+\alpha\left(1-e-\Delta_\sigma^e\right)+\beta\Delta_\sigma^e}=\frac{c}{1+\alpha+(\beta-\alpha)\dfrac{\Delta_\sigma^e}{1-\varepsilon}},$$

for every $e \in \mathcal{E}\backslash\{1\}$, and (2.3) follows.

Next consider the prize game $\tilde{\Gamma}_{\alpha,\beta}^\sigma(P)$. Here the expected utility of agent i, when he chooses effort level e_i and his rival j chooses effort level e_j, is $G^\sigma(e_i-e_j)(1+\beta)P-[1-G^\sigma(e_i-e_j)]\alpha P-ce_i$ (and, if $e_i=e_j$ and $\sigma=0$, replace $G^\sigma(e_i-e_j)$ by $\frac{1}{2}$). Thus, in order to implement (e_1 = 1, e_2 = 1) as a Nash equilibrium of $\tilde{\Gamma}_{\alpha,\beta}^\sigma(P)$, it is necessary and sufficient that P satisfy

$$\frac{1}{2}(1+\beta)P-\frac{1}{2}\alpha P-c\geq G^\sigma(e-1)(1+\beta)P-\left[1-G^\sigma(e-1)\right]\alpha P-ce \tag{6.1}$$

for every $e \in \mathcal{E}\backslash\{1\}$. The minimal P that satisfies (6.1) for every $e_i \in \mathcal{E}$ is thus:

$$\frac{c\max_{e\in\mathcal{E}\backslash\{1\}}\dfrac{1-e}{\dfrac{1}{2}-G^\sigma(e-1)}}{1+\alpha+\beta}.$$

Since G^σ is convex on [–1, 0] and $0 \in \mathcal{E}$, the maximum in this expression is attained for e = 0 (i.e. (6.1) only needs to holds for e = 0), and this leads to (2.4). ■

Proof of Theorem 2:

Since $0\leq\Delta_\sigma\leq\frac{1}{2}$, as remarked in the statement of Proposition 1, (2.6) implies

$$G^\sigma(-1)<\frac{\gamma+(1-\gamma)\Delta_\sigma}{2(1+\gamma)}, \tag{6.2}$$

and thus α^* is well-defined and non-negative. The theorem now follows immediately by comparing (2.3) and (2.4) in Proposition 1. ■

Proof of Theorem 3:

Since $\lim_{\sigma\to 0}G^\sigma(-1)=0$ and $\lim_{\sigma\to 0}\Delta_\sigma^e=0$ for every $e \in \mathcal{E}\backslash\{1\}$, by Proposition 1

$$\lim_{\sigma\to 0}\tilde{M}_{\alpha,\gamma\cdot\alpha}^\sigma=\frac{2c}{1+(1+\gamma)\alpha}<\frac{2c}{1+\alpha}=\lim_{\sigma\to 0}M_{\alpha,\gamma\cdot\alpha}^\sigma. \ ■$$

Proof of Theorem 4:

Since $0 \le \Delta_\sigma \le \frac{1}{2}$, as remarked in the statement of Proposition 1, (2.9) implies

$$G^\sigma(-1) \ge \frac{\gamma + (1-\gamma)\Delta_\sigma}{2(1+\gamma)}. \tag{6.3}$$

By (6.3) and Proposition 1 it then follows that

$$\begin{aligned}
\tilde{M}^\sigma_{\alpha,\gamma\cdot\alpha} &\ge \frac{2(1+\gamma)}{1+(\gamma-1)\Delta_\sigma} \cdot \frac{c}{1+\alpha(\gamma+1)} \\
&= \frac{2}{1+(\gamma-1)\Delta_\sigma} \cdot \frac{c}{\frac{1}{1+\gamma}+\alpha} \\
&\ge \frac{2}{1+(\gamma-1)\Delta_\sigma} \cdot \frac{c}{\frac{1}{1+(\gamma-1)\Delta_\sigma}+\alpha} \\
&= \frac{2c}{1+\alpha+\alpha(\gamma-1)\Delta_\sigma} = M^\sigma_{\alpha,\gamma\cdot\alpha}. \quad \blacksquare
\end{aligned}$$

Proof of Theorem 5:

Fix $0 < \alpha \le \beta$. Suppose to the contrary that there exists a vanishing sequence $\{\sigma_k\}_{k=1}^\infty$ of positive numbers and $(P_k^*, r_k^*) \in \tilde{\Pi}^{\sigma_k}_{\alpha,\beta}$ such that $P_k^* = 0$ (and, w.l.o.g., $r^* \equiv \lim_{k\to\infty} r_k^*$ exists and $0 < r^* < \infty$). Consider $(P^k_{\varepsilon,\delta}, r^k_{\varepsilon,\delta}) = (\varepsilon, r_k^* - \frac{1}{2}\varepsilon(1+\delta))$. We shall show that there exist small enough $\varepsilon > 0$ and $\delta > 0$ such that $(P^k_{\varepsilon,\delta}, r^k_{\varepsilon,\delta}) \in R_+^2$ elicits full effort from both agents in a Nash equilibrium of $\Gamma^{\sigma_k}_{\alpha,\beta}(P^k_{\varepsilon,\delta}, r^k_{\varepsilon,\delta})$ when k is large (and the noise parameter σ_k is small). Since

$$P_k^* + 2r_k^* = 2r_k^* > 2r_k^* - \varepsilon\delta = P^k_{\varepsilon,\delta} + 2r^k_{\varepsilon,\delta},$$

it will follow that $(P_k^*,\ r_k^*)$ are not optimal when k is large, a contradiction.

Now we turn to establishing the existence of the requisite $(P^k_{\varepsilon,\delta},\ r^k_{\varepsilon,\delta})$. First notice that in order to implement $(e_1 = 1,\ e_2 = 1)$ as a Nash equilibrium of $\Gamma^{\sigma_k}_{\alpha,\beta}(P^k_{\varepsilon,\delta}, r^k_{\varepsilon,\delta})$, it is necessary and sufficient for the following incentive conditions to hold:

$$\frac{1}{2}E\left(\left[\begin{array}{c} U\left(P^k_{\varepsilon,\delta} + r^k_{\varepsilon,\delta}\left(1+\varepsilon_i^{\sigma_k}\right)\right) \\ +\beta V\left(P^k_{\varepsilon,\delta} + r^k_{\varepsilon,\delta}\left(\varepsilon_i^{\sigma_k} - \varepsilon_j^{\sigma_k}\right)\right) \end{array}\right] \middle| \varepsilon_i^{\sigma_k} > \varepsilon_j^{\sigma_k}\right)$$

$$+\frac{1}{2}E\left(\left[\begin{array}{c} U\left(r^k_{\varepsilon,\delta}\left(1+\varepsilon_i^{\sigma_k}\right)\right) \\ -\alpha V\left(P^k_{\varepsilon,\delta} + r^k_{\varepsilon,\delta}\left(\varepsilon_j^{\sigma_k} - \varepsilon_i^{\sigma_k}\right)\right) \end{array}\right] \middle| \varepsilon_i^{\sigma_k} < \varepsilon_j^{\sigma_k}\right) - c$$

$$\geq G^{\sigma_k}(e_i-1)\cdot E\left(\left[\begin{array}{c} U\left(P^k_{\varepsilon,\delta}+r^k_{\varepsilon,\delta}\left(e_i+\varepsilon_i^{\sigma_k}\right)\right) \\ +\beta V\left(P^k_{\varepsilon,\delta}+r^k_{\varepsilon,\delta}\left(e_i+\varepsilon_i^{\sigma_k}-1-\varepsilon_j^{\sigma_k}\right)\right)\end{array}\right]\middle| e_i+\varepsilon_i^{\sigma_k}>1+\varepsilon_j^{\sigma_k}\right)$$

$$+\left(1-G^{\sigma_k}(e_i-1)\right)\cdot E\left(\left[\begin{array}{c} U\left(r^k_{\varepsilon,\delta}\left(e_i+\varepsilon_i^{\sigma_k}\right)\right) \\ -\alpha V\left(P^k_{\varepsilon,\delta}+r^k_{\varepsilon,\delta}\left(1+\varepsilon_j^{\sigma_k}-e_i-\varepsilon_i^{\sigma_k}\right)\right)\end{array}\right]\middle| e_i+\varepsilon_i^{\sigma_k}<1+\varepsilon_j^{\sigma_k}\right)-ce_i$$

for every $e_i \in \mathcal{E}\backslash\{1\}$. Denote by $I_k(\varepsilon, \delta, e_i)$ the difference between the LHS and the RHS of the above inequality. Thus, each of the above incentive conditions is equivalent to

$$I_k(\varepsilon,\delta,e_i)\geq 0. \tag{6.4}$$

Observe that the derivative of I_k with respect to ε, evaluated at $\varepsilon = 0$, is given by

$$\frac{1}{2}E\left(\left[\begin{array}{c} U'(r_k^*(1+\varepsilon_i^{\sigma_k}))\left(1-\frac{1}{2}(1+\delta)(1+\varepsilon_i^{\sigma_k})\right) \\ +\beta V'(r_k^*(\varepsilon_i^{\sigma_k}-\varepsilon_j^{\sigma_k}))\left(1-\frac{1}{2}(1+\delta)(\varepsilon_i^{\sigma_k}-\varepsilon_j^{\sigma_k})\right)\end{array}\right]\middle|\varepsilon_i^{\sigma_k}>\varepsilon_j^{\sigma_k}\right)$$

$$+\frac{1}{2}E\left(\left[\begin{array}{c} U'(r_k^*(1+\varepsilon_i^{\sigma_k}))\left(-\frac{1}{2}(1+\delta)(1+\varepsilon_i^{\sigma_k})\right) \\ +\alpha V'(r_k^*(\varepsilon_j^{\sigma_k}-\varepsilon_i^{\sigma_k}))\left(1-\frac{1}{2}(1+\delta)(\varepsilon_j^{\sigma_k}-\varepsilon_i^{\sigma_k})\right)\end{array}\right]\middle|\varepsilon_i^{\sigma_k}<\varepsilon_j^{\sigma_k}\right)-G^{\sigma_k}(e_i-1)$$

$$\cdot E\left(\left[\begin{array}{c} U'(r_k^*(e_i+\varepsilon_i^{\sigma_k}))\left(1-\frac{1}{2}(1+\delta)(e_i+\varepsilon_i^{\sigma_k})\right) \\ +\beta V'(r_k^*(e_i-1+\varepsilon_i^{\sigma_k}-\varepsilon_j^{\sigma_k}))\left(1-\frac{1}{2}(1+\delta)(e_i-1+\varepsilon_i^{\sigma_k}-\varepsilon_j^{\sigma_k})\right)\end{array}\right]\middle|e_i+\varepsilon_i^{\sigma_k}>1+\varepsilon_j^{\sigma_k}\right)$$

$$-\left(1-G^{\sigma_k}(e_i-1)\right)$$

$$\cdot E\left(\left[\begin{array}{c} U'(r_k^*(e_i+\varepsilon_i^{\sigma_k}))\left(-\frac{1}{2}(1+\delta)(e_i+\varepsilon_i^{\sigma_k})\right) \\ -\alpha V'(r_k^*(1-e_i+\varepsilon_j^{\sigma_k}-\varepsilon_i^{\sigma_k}))\left(1-\frac{1}{2}(1+\delta)(1-e_i+\varepsilon_j^{\sigma_k}-\varepsilon_i^{\sigma_k})\right)\end{array}\right]\middle|e_i+\varepsilon_i^{\sigma_k}<1+\varepsilon_j^{\sigma_k}\right),$$

for every $e_i \in \mathcal{E}\backslash\{1\}$. Since the random noises belong to a bounded interval by our assumption in this section, they converge to zero in probability as $\sigma \to 0$. Bearing in mind that $G^{\sigma}(t) \to_{\sigma\to 0} 0$ for $t < 1$ and that U', V' are continuous and bounded, as $k \to \infty$ the above expression converges to:

$$\frac{1}{2}\left(U'(r^*)\left(1-\frac{1}{2}(1+\delta)\right)+\beta V'(0)\right)+\frac{1}{2}\left(U'(r^*)\left(-\frac{1}{2}(1+\delta)\right)-\alpha V'(0)\right)$$

$$-U'(r^*e_i)\left(-\frac{1}{2}(1+\delta)e_i\right)+\alpha V'(r^*(1-e_i))\left(1-\frac{1}{2}(1+\delta)(1-e_i)\right).$$

As $\alpha \le \beta$, the last expression is bounded from below by

$$-\frac{1}{2}U'(r^*)\delta+\alpha V'(r^*(1-e_i))\left(1-\frac{1-e_i}{2}(1+\delta)\right)\ge -\frac{1}{2}B\delta+\alpha b\left(\frac{1}{2}(1-\delta)\right).$$

This is positive for $\delta^* \equiv \dfrac{\alpha b}{2(\alpha b+B)}$, and so $\dfrac{\partial}{\partial \varepsilon}I_k(\varepsilon,\delta^*,e_i)|_{\varepsilon=0}>0$ for all large enough k. Thus, since the incentive constraint (6.4) for any given $e_i \in \mathcal{E}\backslash\{1\}$ holds for $(P_k^*, r_k^*)=(P_{0,\delta^*}^k, r_{0,\delta^*}^k)$, it also holds for $(P_{\varepsilon,\delta^*}^k,\ r_{\varepsilon,\delta^*}^k)$ for all large enough k and some $\varepsilon = \varepsilon(k) > 0$. Since $\mathcal{E}$ is finite, there is only a finite number of incentive constraints, and thus all of them hold simultaneously for $(P_{\varepsilon,\delta^*}^k,\ r_{\varepsilon,\delta^*}^k)$ for all large enough k and some $\varepsilon = \varepsilon^*(k) > 0$. Therefore $(P_{\varepsilon^*(k),\delta^*}^k,\ r_{\varepsilon^*(k),\delta^*}^k)$ elicits full effort from both agents in a Nash equilibrium of $\Gamma_{\alpha,\beta}^{\sigma_k}(P_{\varepsilon^*(k),\delta^*}^k, r_{\varepsilon^*(k),\delta^*}^k)$. As was said, this contradicts the optimality of $(P_k^*,\ r_k^*)$ when k is large. ■

Proof of Theorem 6:

Let $e \in \mathcal{E}\backslash\{1\}$. Denote by r_α^e the minimal piece-rate at which, in the wage game $\Gamma_{\alpha,\gamma\cdot\alpha}^{\sigma}$, effort level 1 is not less attractive to an agent than effort level e, given that his rival chooses effort level 1. Thus r_α^e is the *smallest* among all non-negative numbers r that satisfy the inequality

$$EU\left(r\left(1+\varepsilon_i^\sigma\right)\right)+\gamma\alpha EV\left(\max\left\{r\left(1+\varepsilon_i^\sigma\right)-r\left(1+\varepsilon_j^\sigma\right),0\right\}\right) \tag{6.5a}$$

$$-\alpha EV\left(\max\left\{r\left(1+\varepsilon_j^\sigma\right)-r\left(1+\varepsilon_i^\sigma\right),0\right\}\right)-c \tag{6.5b}$$

$$\ge EU\left(r\left(e+\varepsilon_i^\sigma\right)\right)+\gamma\alpha EV\left(\max\left\{r\left(e+\varepsilon_i^\sigma\right)-r\left(1+\varepsilon_j^\sigma\right),0\right\}\right) \tag{6.5c}$$

$$-\alpha EV\left(\max\left\{r\left(1+\varepsilon_j^\sigma\right)-r\left(e+\varepsilon_i^\sigma\right),0\right\}\right)-ce, \tag{6.5d}$$

or

$$E\left[U\left(r\left(1+\varepsilon_i^\sigma\right)\right)-U\left(r\left(e+\varepsilon_i^\sigma\right)\right)\right] \tag{6.6a}$$

$$+\gamma\alpha E\begin{bmatrix} V\left(\max\left\{r\left(1+\varepsilon_i^\sigma\right)-r\left(1+\varepsilon_j^\sigma\right),0\right\}\right) \\ -V\left(\max\left\{r\left(e+\varepsilon_i^\sigma\right)-r\left(1+\varepsilon_j^\sigma\right),0\right\}\right)\end{bmatrix} \tag{6.6b}$$

$$+\alpha E\begin{bmatrix} V\left(\max\left\{r\left(1+\varepsilon_j^\sigma\right)-r\left(e+\varepsilon_i^\sigma\right),0\right\}\right) \\ -V\left(\max\left\{r\left(1+\varepsilon_j^\sigma\right)-r\left(1+\varepsilon_i^\sigma\right),0\right\}\right)\end{bmatrix} \tag{6.6c}$$

$$\geq c(1-e). \tag{6.6d}$$

Let

$$K \equiv \min_{0\leq x\leq 1+2\lambda} V'(x) > 0.$$

Then, for all $r \leq 1$

$$E\left[U\left(r\left(1+\varepsilon_i^\sigma\right)\right)-U\left(r\left(e+\varepsilon_i^\sigma\right)\right)\right] \tag{6.7a}$$

$$+\gamma\alpha E\begin{bmatrix} V\left(\max\left\{r\left(1+\varepsilon_i^\sigma\right)-r\left(1+\varepsilon_j^\sigma\right),0\right\}\right) \\ -V\left(\max\left\{r\left(e+\varepsilon_i^\sigma\right)-r\left(1+\varepsilon_j^\sigma\right),0\right\}\right)\end{bmatrix} \tag{6.7b}$$

$$+\alpha E\begin{bmatrix} V\left(\max\left\{r\left(1+\varepsilon_j^\sigma\right)-r\left(e+\varepsilon_i^\sigma\right),0\right\}\right) \\ -V\left(\max\left\{r\left(1+\varepsilon_j^\sigma\right)-r\left(1+\varepsilon_i^\sigma\right),0\right\}\right)\end{bmatrix} \tag{6.7c}$$

$$\geq \alpha E\begin{bmatrix} V\left(\max\left\{r\left(1+\varepsilon_j^\sigma\right)-r\left(e+\varepsilon_i^\sigma\right),0\right\}\right) \\ -V\left(\max\left\{r\left(1+\varepsilon_j^\sigma\right)-r\left(1+\varepsilon_i^\sigma\right),0\right\}\right)\end{bmatrix} \tag{6.7d}$$

$$\geq \alpha\frac{1}{2}Kr(1-e). \tag{6.7e}$$

Consequently, for all large enough α, substituting $r = \dfrac{c}{\frac{1}{2}\alpha K(1-e)} \leq 1$ into (6.5) turns it into a valid inequality by (6.7), and hence $r_\alpha^e \leq \dfrac{c}{\frac{1}{2}\alpha K(1-e)}$ (in particular, $\lim_{\alpha\to\infty} r_\alpha^e = 0$). Substituting $r = r_\alpha^e$ into (6.6), we can therefore use the first-order (linear) approximation $U'(0)\cdot x$ for $U(x)$, and $V'(0)\cdot x$ for $V(x)$, around 0, to derive an existence of $\tau_\alpha^e \geq 0$ such that

$$U'(0)\cdot r_\alpha^e(1-e)$$

$$+\gamma\alpha V'(0)\cdot r_\alpha^e\left(E\left[\max\left\{1+\varepsilon_i^\sigma-1-\varepsilon_j^\sigma,0\right\}\right]-E\left[\max\left\{e+\varepsilon_i^\sigma-1-\varepsilon_j^\sigma,0\right\}\right]\right)$$

$$+\alpha V'(0)\cdot r_\alpha^e\left(E\left[\max\left\{1+\varepsilon_j^\sigma-e-\varepsilon_i^\sigma,0\right\}\right]-E\left[\max\left\{1+\varepsilon_j^\sigma-1-\varepsilon_i^\sigma,0\right\}\right]\right)$$

$$\geq c(1-e)-\tau_\alpha^e$$

holds for every α, and $\lim_{\alpha\to\infty}\tau_\alpha^e = 0$. Using the definition of Δ_σ^e in (2.1), this can be rewritten as

$$\left(U'(0)+\alpha V'(0)+(\gamma-1)\alpha V'(0)\frac{\Delta_\sigma^e}{1-e}\right)\cdot r_\alpha^e \geq c-\frac{\tau_\alpha^e}{1-e},$$

or

$$r_\alpha^e \geq \frac{c-\dfrac{\tau_\alpha^e}{1-e}}{U'(0)+\alpha V'(0)+(\gamma-1)\alpha V'(0)\dfrac{\Delta_\sigma^e}{1-e}}.$$

The minimal piece rate that implements (e_1 = 1, e_2 = 1) as a Nash equilibrium in the wage game $\Gamma_{\alpha,\gamma\cdot\alpha}^\sigma$ should therefore be at least

$$\frac{c-\tau_\alpha}{U'(0)+\alpha V'(0)+(\gamma-1)\alpha V'(0)\Delta_\sigma},$$

for Δ_σ defined in (2.2) and $\tau_\alpha \equiv \max_{e\in\mathcal{E}\setminus\{1\}}\dfrac{\tau_\alpha^e}{1-e}$. Consequently,

$$M_{\alpha,\gamma\cdot\alpha}^\sigma \geq \frac{2(c-\tau_\alpha)}{U'(0)+\alpha V'(0)+(\gamma-1)\alpha V'(0)\Delta_\sigma} \tag{6.8}$$

for all sufficiently large α.

Arguing as in the end of the proof of Proposition 1, one can show that the minimal prize $\tilde{M}_{\alpha,\gamma\cdot\alpha}^\sigma$ that implements (e_1 = 1, e_2 = 1) as a Nash equilibrium in the prize game $\tilde{\Gamma}_{\alpha,\gamma\alpha}^\sigma$ satisfies

$$\frac{1}{2}\left(U\left(\tilde{M}_{\alpha,\gamma\cdot\alpha}^\sigma\right)+\gamma\alpha V\left(\tilde{M}_{\alpha,\gamma\cdot\alpha}^\sigma\right)\right)-\frac{1}{2}\alpha V\left(\tilde{M}_{\alpha,\gamma\cdot\alpha}^\sigma\right)-c \tag{6.9}$$

$$=G^\sigma(-1)\left(U\left(\tilde{M}_{\alpha,\gamma\cdot\alpha}^\sigma\right)+\gamma\alpha V\left(\tilde{M}_{\alpha,\gamma\cdot\alpha}^\sigma\right)\right)-\left[1-G^\sigma(-1)\right]\alpha V\left(\tilde{M}_{\alpha,\gamma\cdot\alpha}^\sigma\right). \tag{6.10}$$

It follows that $\tilde{M}_{\alpha,\gamma\cdot\alpha}^\sigma = F_\alpha^{-1}\left(\dfrac{c}{\dfrac{1}{2}-G^\sigma(-1)}\right)$, where[19] $F_\alpha(x) \equiv U(x) + (1+\gamma)\alpha V(x)$. Since $(1+\gamma)\alpha V \leq F_\alpha$ on $\mathbb{R}_+$,

[19] Since U and V are strictly increasing, F_α is invertible.

$$\tilde{M}^{\sigma}_{\alpha,\gamma\cdot\alpha} \le V^{-1}\left(\frac{1}{(1+\gamma)\alpha}\cdot\frac{c}{\frac{1}{2}-G^{\sigma}(-1)}\right) \le \frac{\tilde{K}}{\alpha}$$

for some $\tilde{K} > 0$ and for all large enough α (and in particular $\lim_{\alpha\to\infty}\tilde{M}^{\sigma}_{\alpha,\gamma\cdot\alpha} = 0$). We can therefore use (6.9) and the linear approximation $U'(0)\cdot x$ for $U(x)$, and $V'(0)\cdot x$ for $V(x)$, around 0, to derive the existence of a $\delta_\alpha \ge 0$ such that

$$\frac{1}{2}(U'(0)+\gamma\alpha V'(0))\cdot\tilde{M}^{\sigma}_{\alpha,\gamma\cdot\alpha} - \frac{1}{2}(\alpha V'(0))\cdot\tilde{M}^{\sigma}_{\alpha,\gamma\cdot\alpha} - c - \delta_\alpha$$

$$\le G^{\sigma}(-1)(U'(0)+\gamma\alpha V'(0))\cdot\tilde{M}^{\sigma}_{\alpha,\gamma\cdot\alpha} - \left[1-G^{\sigma}(-1)\right]\alpha V'(0)\cdot\tilde{M}^{\sigma}_{\alpha,\gamma\cdot\alpha}$$

holds for every α, and $\lim_{\alpha\to\infty}\delta_\alpha = 0$. Thus

$$\tilde{M}^{\sigma}_{\alpha,\gamma\cdot\alpha} \le \frac{c+\delta_\alpha}{(U'(0)+(1+\gamma)\alpha V'(0))}\cdot\frac{1}{\frac{1}{2}-G^{\sigma}(-1)} \tag{6.11}$$

for all sufficiently large α.

It follows from (6.8) and (6.11) that

$$\limsup_{\alpha\to\infty}\frac{\tilde{M}^{\sigma}_{\alpha,\gamma\cdot\alpha}}{M^{\sigma}_{\alpha,\gamma\cdot\alpha}} \le \frac{1+(\gamma-1)\Delta_\sigma}{2(1+\gamma)}\cdot\frac{1}{\frac{1}{2}-G^{\sigma}(-1)} < 1,$$

where the last inequality holds by (2.6),[20] and thus indeed $\tilde{M}^{\sigma}_{\alpha,\gamma\cdot\alpha} < M^{\sigma}_{\alpha,\gamma\cdot\alpha}$ for all sufficiently large α. ■

Proof of Theorem 7:

Suppose that the assertion is false for some $\alpha \ge \beta > 0$. Then one can find two non-negative sequences, $(\sigma_k)_{k=1}^{\infty}$ with $\lim_{k\to\infty}\sigma_k = 0$, and a sequence $(w_k)_{k=1}^{\infty}$ of wage contracts, such that

$$\bar{M}^{\sigma_k}_{\alpha,\beta}(w_k) \le \tilde{M}^{\sigma_k}_{\alpha,\beta} \tag{6.12}$$

for all k, and w_k implements maximal effort by both agents in Nash equilibrium when agents' outputs are affected by noises $\varepsilon_1^{\sigma_k}$, $\varepsilon_2^{\sigma_k}$. From (2.3), (2.4) and the fact that $G^{\sigma_k}(-1)\to 0$ as $\sigma_k \to 0$, we obtain

$$\tilde{M}^{\sigma_k}_{\alpha,\beta} = \frac{c}{\frac{1}{2}-G^{\sigma_k}(-1)}\cdot\frac{1}{1+\alpha+\beta} \to_{k\to\infty} \frac{2c}{1+\alpha+\beta} = \tilde{M}^{0}_{\alpha,\beta}. \tag{6.13}$$

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[20] Or, more precisely, by (6.2), as was explained in the proof of Theorem 2.

On the other hand, we claim that

$$\liminf_{k\to\infty} \bar{M}^{\sigma_k}_{\alpha,\beta}(w_k) \geq \bar{M}^{0}_{\alpha,\beta}. \tag{6.14}$$

Indeed, there is a subsequence of $(w_k)_{k=1}^{\infty}$ (which w.l.o.g. is taken to be the sequence itself) such that the limit

$$\bar{r} \equiv \lim_{k\to\infty} Ew_k\left(1+\varepsilon_1^{\sigma_k}\right) = \frac{1}{2}\liminf_{k\to\infty} \bar{M}^{\sigma_k}_{\alpha,\beta}(w_k) \tag{6.15}$$

exists.[21] Since agent i prefers $e_i = 1$ to $e_i = 0$ given $e_j = 1$ when there is noise $\varepsilon_i^{\sigma_k}$ that affects his (and independently his opponent's) output under wage function w_k,

$$E\left(w_k\left(1+\varepsilon_i^{\sigma_k}\right)\right)+\beta E\left(\max\left\{w_k\left(1+\varepsilon_i^{\sigma_k}\right)-w_k\left(1+\varepsilon_j^{\sigma_k}\right),0\right\}\right)$$

$$-\alpha E\left(\max\left\{w_k\left(1+\varepsilon_j^{\sigma_k}\right)-w_k\left(1+\varepsilon_i^{\sigma_k}\right),0\right\}\right)-c$$

$$\geq E\left(w_k\left(\varepsilon_i^{\sigma_k}\right)\right)+\beta E\left(\max\left\{w_k\left(\varepsilon_i^{\sigma_k}\right)-w_k\left(1+\varepsilon_j^{\sigma_k}\right),0\right\}\right)$$

$$-\alpha E\left(\max\left\{w_k\left(1+\varepsilon_j^{\sigma_k}\right)-w_k\left(\varepsilon_i^{\sigma_k}\right),0\right\}\right).$$

But $E\left(w_k\left(\varepsilon_i^{\sigma_k}\right)\right)\geq 0$ and $\beta \leq \alpha$, and hence it follows that

$$E\left(w_k\left(1+\varepsilon_i^{\sigma_k}\right)\right)-c \tag{6.16}$$

$$\geq -\alpha E\left(w_k\left(1+\varepsilon_j^{\sigma_k}\right)\right)+(\beta-\alpha)E\left(\max\left\{w_k\left(\varepsilon_i^{\sigma_k}\right)-w_k\left(1+\varepsilon_j^{\sigma_k}\right),0\right\}\right). \tag{6.17}$$

Note that as the functions $(w_k)_{k=1}^{\infty}$ are uniformly bounded and $\lim_{k\to\infty} G^{\sigma_k}(-1)=0$,

$$\lim_{k\to\infty} E\left(\max\left\{w_k\left(\varepsilon_i^{\sigma_k}\right)-w_k\left(1+\varepsilon_j^{\sigma_k}\right),0\right\}\right)=0.$$

Thus, taking the limit as $k \to \infty$ of both sides of (6.16)–(6.17) yields

$$\bar{r}-c \geq -\alpha\bar{r} \tag{6.18}$$

for $\bar{r}$ defined in (6.15). Accordingly, agent i would prefer $e_i = 1$ to $e_i = 0$ (or any other effort level) under piece rate $\bar{r}$ when there is no noise. This shows that

$$\lim_{k\to\infty} \bar{M}^{\sigma_k}_{\alpha,\beta}(w_k) = 2\bar{r} \geq \bar{M}^{0}_{\alpha,\beta},$$

and establishes (6.14).

Now the combination of (6.12), (6.13), and (6.14) contradicts (4.1), which proves the theorem. ■

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[21] Note that $\bar{r} < \infty$ since the wage functions are uniformly bounded.

Proof of Theorem 9:

Assume, as in Section 5, that random variables $\left(\varepsilon_k^\sigma\right)_{k=1}^\infty$ have bounded support $[-\sigma, \sigma]$ and possess a continuously differentiable and strictly positive density function f^σ on it.

Lemma 1: *There exists $N > 0$ such that the cumulative distribution function G_n^σ of the random variable $\left(\max_{1\le j\le n, j\ne i}\varepsilon_j^\sigma\right) - \varepsilon_i^\sigma$ is convex on* $(-\infty, 0]$ provided $n \geq N$.

Proof: For every $-2\sigma \leq t \leq 0$, G_n^σ is given by

$$G_n^\sigma(t) = \int_{-\sigma-t}^{\sigma} \Pr\left(\max_{1\le j\le n, j\ne i} \varepsilon_j^\sigma - \varepsilon_i^\sigma \le t \mid \varepsilon_i^\sigma = y\right) f^\sigma(y)\,dy = \int_{-\sigma-t}^{\sigma} F^\sigma(y+t)^{n-1} f^\sigma(y)\,dy, \quad (6.19)$$

where f^σ denotes the density function of each ε_j^σ. Using (6.19),

$$\frac{\partial}{\partial t} G_n^\sigma(t) = \int_{-\sigma-t}^{\sigma} (n-1) F^\sigma(y+t)^{n-2} \frac{\partial}{\partial t} F^\sigma(y+t) f^\sigma(y)\,dy$$

$$+F^\sigma((-\sigma-t)+t)^{n-1} f^\sigma(-\sigma-t) = \int_{-\sigma-t}^{\sigma} (n-1) F^\sigma(y+t)^{n-2} f^\sigma(y+t) f^\sigma(y)\,dy,$$

and (for $n \geq 3$)

$$\frac{\partial}{\partial^2 t} G_n^\sigma(t) = \int_{-\sigma-t}^{\sigma} (n-1)(n-2) F^\sigma(y+t)^{n-3} f^\sigma(y+t)^2 f^\sigma(y)\,dy$$

$$+\int_{-\sigma-t}^{\sigma} (n-1) F^\sigma(y+t)^{n-2} \frac{\partial}{\partial t} f^\sigma(y+t) f^\sigma(y)\,dy$$

$$+(n-1) F^\sigma((-\sigma-t)+t)^{n-2} f^\sigma((-\sigma-t)+t) f^\sigma(-\sigma-t)$$

$$= (n-1)\int_{-\sigma-t}^{\sigma} F^\sigma(y+t)^{n-3} \left[(n-2) f^\sigma(y+t)^2 + F^\sigma(y+t) \frac{\partial}{\partial t} f^\sigma(y+t)\right] f^\sigma(y)\,dy.$$

Since $\min_{y\in[-\sigma,\sigma]} f^\sigma(y) > 0$, it is clear that

$$(n-2) f^\sigma(y+t)^2 + F^\sigma(y+t) \frac{\partial}{\partial t} f^\sigma(y+t) > 0$$

for every $y \in [-\sigma - t, \sigma]$ and for all sufficiently large n. We conclude that $\frac{\partial}{\partial^2 t} G_n^\sigma(t) > 0$ and thus the function G_n^σ is convex on $[-2\sigma, 0]$ for all sufficiently large n. Since $G_n^\sigma \equiv 0$ on $(-\infty, -2\sigma]$, G_n^σ is in fact convex on the entire $[-\infty, 0]$. ■

Lemma 2: $\lim_{n\to\infty} n^2 G_n^\sigma(-1) = 0$.

Proof: Using (6.19) in the proof of Lemma 1,

$$G_n^\sigma(-1) \le F^\sigma(\sigma-1)^{n-1}$$

if $2\sigma \geq 1$, and $G_n^\sigma(-1) = 0$ otherwise. Since $F^\sigma(\sigma - 1) < 1$, obviously

$$\lim_{n\to\infty} n^2 G_n^\sigma(-1) = 0. \blacksquare$$

Theorem 9 now follows immediately from (5.3), given (5.1). ■

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