ENDOGENOUS COLLATERAL CONSTRAINTS
AND THE LEVERAGE CYCLE

By

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Endogenous Collateral Constraints and the Leverage Cycle

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Abstract

We review the theory of leverage developed in collateral equilibrium models with incomplete markets. We explain how leverage tends to boost asset prices and create bubbles. We show how leverage can be endogenously determined in equilibrium and how it depends on volatility. We describe the dynamic feedback properties of leverage, volatility, and asset prices, in what we call the leverage cycle, and show how it differs from a credit cycle. We also describe some cross-sectional implications of multiple leverage cycles, including contagion, flight to collateral, and swings in the issuance volume of the highest-quality debt.

1. INTRODUCTION

During the Great Moderation of the late 1990s and up through 2006, volatility was low, while debt issuance and asset prices soared. During the crisis of 2007–2009 and its aftermath, now called the Great Recession, volatility was high, while private debt issuance and asset prices plummeted. Before the financial crisis of 2007–2009, mainstream macroeconomics assigned little if any role to financial frictions in explaining aggregate fluctuations (see, e.g., Smets & Wouters 2007). The recent financial crisis, however, has challenged this view. It is now widely recognized that financial factors were central to the recent crisis, and as a result, their role in explaining economic fluctuations is being reconsidered.

Collateral general equilibrium theory explains the connection among volatility, debt issuance, and asset prices through a ratio called leverage. The idea is that supply and demand determine how much collateral backs each promise, which is a ratio. The higher the future volatility of the collateral values, the more collateral will be required by lenders to feel secure. Leverage can be measured in many equivalent ways; we focus on the value of the loan divided by the value of the collateral, the loan to value (LTV).[1] The recent economic turmoil has brought to the forefront the role of leverage as an important driver of asset prices and economic activity. During the Great Moderation, leverage also soared, and it also plummeted during the crisis of 2007–2009 and its aftermath (for accounts of this, see, e.g., Brunnermeier 2009, Geanakoplos 2010, Gorton 2009).

The purpose of this article is to review the leverage cycle theory derived from collateral equilibrium models in Geanakoplos (1997, 2003), and extended to multiple leverage cycles in Fostel & Geanakoplos (2008), before the financial crisis.[2] This article describes parts of these papers through a sequence of simple variations of one baseline example.

The leverage cycle can be described as follows. Lenders do not trust borrowers' promises to repay. They insist on collateral, which constrains how much people can borrow: Agents cannot borrow more at the going interest rates if they do not have the collateral. When volatility is low for an extended period of time, leverage rises, both because lenders feel more secure and because Wall Street innovates to stretch the available scarce collateral. As shown in **Figure 1**, at the beginning of the Great Moderation, borrowing $86 or less on a $100 house was normal. By the end of the Great Moderation, leverage had risen so much that by late 2006, it was normal to borrow $97 on a $100 house.

When leverage rises throughout the economy, and not just for one borrower, collateralizable asset prices rise. More people can afford to buy, buyers can purchase more units, and they are willing to spend more for the collateral because they can use it to borrow. Borrowing therefore rises with leverage for compounded reasons: It is a higher percentage (higher LTV) of a higher number (higher collateral prices). At the ebullient stage, when leverage is at its highest, the economy appears to be in wonderful shape: Prices and investor's profits are high and stable, and economic activity is booming.

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[1]Leverage is also measured as the ratio of the value of the collateral to the cash down payment used to buy it, or sometimes as the ratio of debt to equity (which at origination is the down payment). When we speak of leverage, we always mean on new debt. Reinhart & Rogoff (2009) find that leverage (in their definition, the ratio of debt to GDP) continues to rise for several years after financial crises. But that is because they are measuring outstanding debt, which is mostly old debt. They would have found that leverage on new debt falls after financial crises.

[2]Starting in the 1970s, Hyman Minsky wrote about a disequilibrium cycle he called the instability principle, which he linked to leverage (see, e.g., Minsky 1986). He envisaged periods of inflation and deflation, tracing part of his ideas back to Irving Fisher's famous debt deflation. He did not suggest that the inflation would come foremost in collateral goods. His key concept was the transition from promising no more than future income flows to borrowing beyond that, which he called speculative finance or Ponzi finance. He did not present a theory for what determined leverage (except possibly exuberance), nor did he envisage a central role for uncertainty or volatility. By contrast, the leverage cycle is an equilibrium theory in which changes in volatility, endogenous leverage, and collateral prices play the central roles.
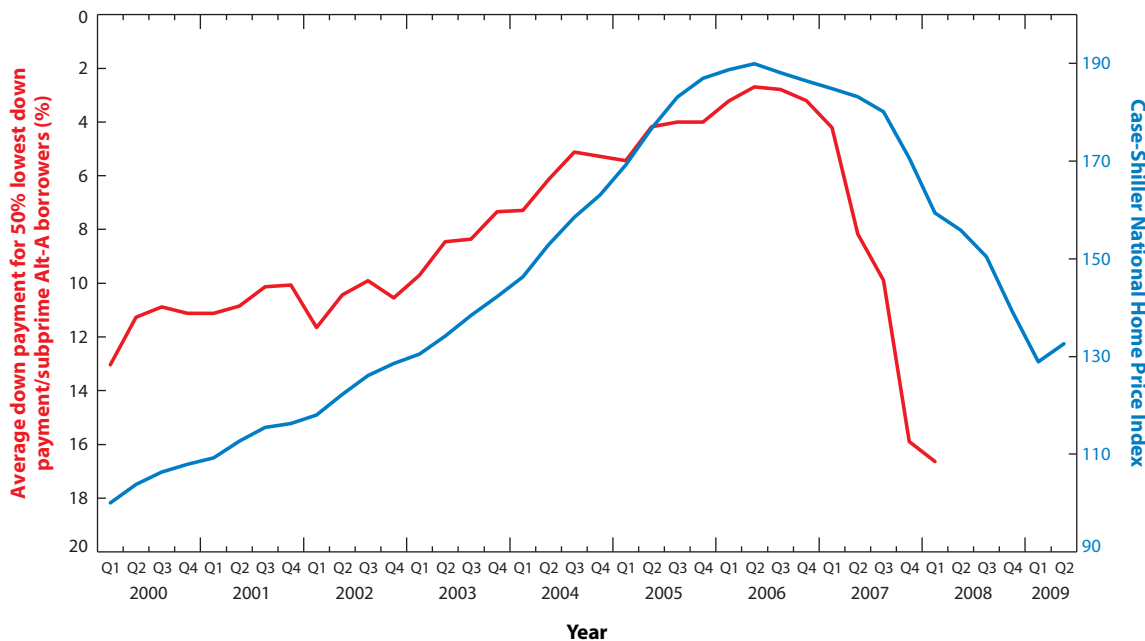
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Figure 1

Housing leverage cycle, showing the margins offered (down payments required) and housing prices in the United States from 2000 to 2009. Observe that the down-payment axis has been reversed because lower down-payment requirements are correlated with higher home prices. For every Alt-A or subprime first loan originated from Q1 2000 to Q2 2008, the down-payment percentage was calculated as the appraised value (or sale price if available) minus the total mortgage debt, divided by the appraised value. (Subprime/Alt-A issuance stopped in Q1 2008.) For each quarter, the down-payment percentages were ranked from highest to lowest, and the average of the bottom half of the list is shown in the figure. This number is an indicator of the down payment required. Clearly many homeowners put down more than they had to, and this is why the top half of the list is dropped from the average. A 13% down payment in Q1 2000 corresponds to leverage of about 7.7, and a 2.7% down payment in Q2 2006 corresponds to leverage of about 37. **Figure 1** reproduced from Geanakoplos (2010).

However, this is precisely the phase at which the economy is most vulnerable. A little bit of bad news that causes asset prices to fall has a big impact on the most enthusiastic and biggest buyers because they are the most leveraged.[3] Most importantly, if the bad news increases uncertainty or volatility, lenders will tighten credit. In 2006, the $2.5 trillion of so-called toxic mortgage securities that later threatened the whole financial system could have been purchased with a down payment of approximately $150 billion, with the remaining $2.35 trillion spent out of borrowed money (LTV of 93%). In 2008, those same securities required a down payment of 75%; at 2006 prices, that would have meant a down payment of almost $2 trillion cash, and just $600 billion borrowed (LTV of 25%). Within two years, leverage for these assets fell from approximately 16 to less than 1.4. As **Figure 1** shows, the normal down payment for housing financed by nongovernment mortgages fell from 13% in 2000 to 2.7% in 2006, and then rose to 16% in 2007.

As shown below, leverage cycle crashes always occur because of a coincidence of three factors. The bad news itself lowers the prices. But it also drastically reduces the wealth of the leveraged buyers, who were leveraged the most precisely because they are the most optimistic buyers. Thus, the purchasing power of the most willing buyers is reduced. And most importantly, if the bad news

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[3]Investors who are leveraged 30 to 1 lose 30% of their investments when the asset price falls only 1%.

also creates more uncertainty and volatility, then credit markets tighten and leverage will be reduced, just when the optimists would like to borrow more, making it much harder for the optimists to retain their assets in the face of margin calls, and making it much harder for any potential new buyers to find funding to purchase the forced sales of assets.

There is a growing literature on leverage. Some papers focus on investor-based leverage, which can be measured by the ratio of total debt to total equity in an investor's portfolio (e.g., Acharya & Viswanathan 2011, Adrian & Shin 2010, Brunnermeier & Sannikov 2014, Gromb & Vayanos 2002). Other papers focus on asset-based leverage as defined above (e.g., Acharya et al. 2011; Brunnermeier & Pedersen 2009; Fostel & Geanakoplos 2008, 2012a,b, 2013; Geanakoplos 1997, 2003, 2010; Simsek 2013).

Not all these models actually make room for endogenous leverage. Often an ad hoc behavioral rule is postulated. To mention just a few, Brunnermeier & Pedersen (2009) assume a Value at Risk (VaR) rule, which limits borrowing so that the probability of default cannot exceed an exogenously set parameter such as 5%. Gromb & Vayanos (2002) and Vayanos & Wang (2012) assume a maxmin rule that prevents default altogether. Some other papers assume a fixed LTV (e.g., Garleanu & Pedersen 2011, Mendoza 2010). In some papers (e.g., Brunnermeier & Sannikov 2014), leverage is endogenous, but borrowers are not constrained: They are borrowing all they would like to at the going riskless rate of interest but become more cautious if the world grows more uncertain.

The leverage cycle theory reviewed in this article models the effect of leverage on asset prices and economic activity and also provides a model of the endogenous determination of collateral requirements. At first glance, this seems a difficult problem: How can one supply-equals-demand equation for loans determine two variables, the interest rate and the LTV? In collateral equilibrium models developed by Geanakoplos (1997) and Geanakoplos & Zame (1997), the puzzle is solved by postulating that equilibrium prices consist of a menu, with a different interest rate for each LTV. Geanakoplos (1997, 2003) shows that in some special cases, all agents would choose the same contract from the menu. Fostel & Geanakoplos (2013) prove that in all binomial economies with financial assets, one and only one contract is indeed chosen. We review these findings in Sections 2.1–2.4.

Collateral equilibrium models also provide a framework to study the asset pricing implications of leverage. A key point from Geanakoplos (1997) is that in a collateral equilibrium, investors do not always set the marginal utility of an asset's dividends equal to its price; rather, they set the marginal utility of the asset's dividends net of the loan repayments equal to its down payment. Based on this insight, Fostel & Geanakoplos (2008) develop a formal theory of asset pricing that links liquidity and collateral to asset prices. Collateralizable assets always trade at a price equal to their payoff value plus a nonnegative collateral value because they enable their holders to borrow money. We review these findings in Sections 2.5–2.8.

In Section 3, we review the leverage cycle of Geanakoplos (2003), in which exogenous changes in volatility create endogenous changes in leverage that move asset prices much more than any agent thinks is warranted by the news alone. In Section 3.11 and in **Supplemental Appendix 4** (follow the Supplemental Material link from the Annual Reviews home page at **http://www. annualreviews.org**), we describe the agent-based behavioral model of Thurner et al. (2012), in which small independently and identically distributed noise creates large changes in asset prices as the result of changing leverage and margin calls.

In Section 4, we review how Fostel & Geanakoplos (2008) extend the leverage cycle model to include multiple leverage cycles over different asset classes. Collateral equilibrium pricing theory is used to explain cross-sectional properties such as flight to collateral, contagion, and the enormous volatility in the volume of trade of high-quality assets.

Finally, let us mention that before the crisis, another branch of nonmainstream macroeconomics, led by Bernanke & Gertler (1989) and Kiyotaki & Moore (1997), also investigated

collateral and what Kiyotaki & Moore (1997) call the credit cycle. The credit cycle featured the multiplier-accelerator feedback from good news about asset dividends, to higher asset prices, to more borrowing, to more investments improving asset values. Nevertheless, the credit cycle literature missed some important elements of the leverage cycle. The credit cycle ignored leverage. The multiplier-accelerator story would work with a constant LTV; in fact, in Kiyotaki & Moore (1997), leverage falls when asset prices are rising, dampening the cycle instead of driving it. Volatility plays no role in the credit cycle. Endogenous leverage, and certainly changing leverage, is not really a focus of the credit cycle models.

To the extent that endogenous leverage was considered at all, it was in a corporate finance context, in which assets cannot be fully leveraged because lenders want to see that the borrowers have skin in the game to incentivize them to work harder to improve the dividends of the assets (as in Holmstrom & Tirole 1997). However, the holders of mortgage securities (and to a great extent the owners of houses), which formed the bulk of the collateral that fueled the 2000–2009 leverage cycle, had no control over the dividends or value of those securities. Leverage changed as lenders (perhaps led or misled by rating agencies) got more or less nervous about the future value of the assets. Finally, the credit cycle literature emphasized the view that collateral constraints depress the value of assets and prevent investors from finding the money to invest as much as they wish. But that misses the collateral value of assets. When the only way to borrow is by holding certain kinds of collateral, the good collateral will rise, not fall, in price, leading to overinvestment and even bubbles. The credit cycle literature missed the bubble of the leverage cycle, as well as the speedy collapse brought on by rapidly falling leverage.

2. A BINOMIAL MODEL OF ENDOGENOUS LEVERAGE

2.1. The Model

In this section, we present the collateral general equilibrium model and the main theoretical results concerning leverage, liquidity, and asset prices.

2.1.1. Time and uncertainty. Consider a finite-horizon general equilibrium model, with time $t = 0, \ldots, T$. Uncertainty is represented by a binomial tree of date events or states $s \in S$, including a root $s = 0$. Each state $s \neq 0$ has an immediate predecessor s^*, and each nonterminal node $s \in S \backslash S_T$ has a set $S(s) = \{sU, sD\}$ of immediate successors. We denote the time of s by the number of nodes $t(s)$ on the path $(0, s]$ from 0 to s, not including 0. We stick with binomial trees in this review because they are the simplest models in which one can study the role of uncertainty in shaping leverage and because one can prove general theorems about leverage and default in such models.

2.1.2. Goods and assets. There is a single perishable consumption good c and $K = \{1, \ldots, K\}$ assets k, which pay dividends d_s^k of the consumption good in each state $s \in S \backslash \{0\}$. The dividends d_s^k are distributed at state s to the investors who owned the asset in state s^*. We take the consumption good as the numéraire in every state, and $\mathbf{p}_s \in R_+^K$ denotes the vector of asset prices in state s.

We assume that all assets are financial assets; that is, they give no direct utility to investors and pay the same dividends no matter who owns them. Financial assets are valued exclusively because they pay dividends. Houses are not financial assets because they give utility to their owners. Nor is land if its output depends on who owns it and tills it.

2.1.3. Debt and collateral. The heart of our analysis involves contracts and collateral. In an Arrow-Debreu equilibrium, the question of why agents repay their loans is ignored. We suppose from now on that the only enforcement mechanism is collateral.

A debt contract $j \in J$ is a one-period noncontingent bond issued in state $s(j) \in S$ that promises $b(j) > 0$ units of the consumption good in each immediate successor state $s' \in S(s)$, using one unit of asset $k(j) \in K$ as collateral. We denote the set of contracts with issue state s backed by one unit of asset k by $J_s^k \subset J$; we let $J_s = \bigcup_k J_s^k$ and $J = \bigcup_{s \in S \setminus S_T} J_s$.

The price of contract j in state $s(j)$ is π_j. An investor can borrow π_j at $s(j)$ by selling contract j, promising $b(j)$ in each $s' \in S(s(j))$, provided he or she holds one unit of asset $k(j)$ as collateral. Let φ_j be the number of contracts j traded at $s(j)$. There is no sign constraint on φ_j. A positive φ_j indicates the agent is selling $|\varphi_j|$ contracts j or borrowing $|\varphi_j| \pi_j$; a negative φ_j indicates the agent is buying $|\varphi_j|$ contracts j or lending $|\varphi_j| \pi_j$.

We assume that the most borrowers can lose is their collateral if they do not honor their promise, as is the case with no-recourse collateral. Hence, the actual delivery of contract j in each state $s' \in S(s(j))$ is

$$\min\left\{ b(j), p_{s'k(j)} + d_{s'}^{k(j)} \right\}. \tag{1}$$

The rate of interest promised by contract j in equilibrium is $(1 + r_j) = b(j)/\pi_j$. If the promise is small enough that $b(j) \leq p_{s'k(j)} + d_{s'}^{k(j)}, \forall s' \in S(s(j))$, then the contract will not default. In this case, its price defines a riskless rate of interest. In equilibrium, all one-period contracts j that do not default and are issued at the same state $s = s(j)$ can be priced so as to define the same riskless rate of interest, which we call r_s.

The LTV_j associated with contract j in state $s(j)$ is given by

$$\mathrm{LTV}_j = \frac{\pi_j}{p_{s(j)k(j)}}. \tag{2}$$

The margin or down-payment rate m_j associated with contract j in state $s(j)$ is $1 - \mathrm{LTV}_j$. Leverage associated with contract j in state $s(j)$ is the inverse of the margin, $1/m_j$, and moves monotonically with LTV_j.

We define leverage for asset k in state s, LTV_s^k, as the trade-value weighted average of LTV_j across all actively traded debt contracts $j \in J_s^k$ by all the agents $h \in H$:

$$\mathrm{LTV}_s^k = \frac{\sum_h \sum_{j \in J_s^k} \max\left(0, \varphi_j^h\right) \pi_j}{\sum_h \sum_{j \in J_s^k} \max\left(0, \varphi_j^h\right) p_{sk}}. \tag{3}$$

Finally, leverage for investor h in state s, LTV_s^h, is defined analogously as

$$\mathrm{LTV}_s^h = \frac{\sum_k \sum_{j \in J_s^k} \max\left(0, \varphi_j^h\right) \pi_j}{\sum_k \sum_{j \in J_s^k} \max\left(0, \varphi_j^h\right) p_{sk}}. \tag{4}$$

2.1.4. Investors. Each investor $h \in H$ is characterized by a utility, u^h, a discount factor, δ_h, and subjective probabilities, γ_s^h, denoting the probability of reaching state s from its predecessor s^*, for all $s \in S \setminus \{0\}$.[4] We assume that the utility function for consumption in each state $s \in S, u^h : R_+ \to R$, is differentiable, concave, and monotonic. The expected utility to agent h is

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[4]Of course, $\gamma_{sD}^h = 1 - \gamma_{sU}^h, \forall s \in S \setminus S_T$.

$$U^h = u^h(c_0) + \sum_{s \in S \setminus 0} \delta_h^{t(s)} \overline{\gamma}_s^h u^h(c_s), \tag{5}$$

where $\overline{\gamma}_s^h$ is the probability of reaching s from 0 (obtained by taking the product of γ_σ^h over all nodes σ on the path $(0, s]$ from 0 to s).

Investor h's endowment of the consumption good is denoted by e_s^h in each state $s \in S$. His or her endowment of the assets in state s is $\mathbf{a}_s^h \in R_+^K$, and it entitles the investor to the dividends $\mathbf{d}_s \cdot \mathbf{a}_s^h$ and the right to subsequently sell those assets in s. We assume that the consumption good is always present, $\sum_{h \in H} \left(e_s^h + \mathbf{d}_s \cdot \sum_{\{\sigma : \sigma \leq s\}} \mathbf{a}_s^h \right) > 0, \forall s \in S$. We suppose agents start with no debts, $J_{0^*} = \emptyset$.

Given asset prices and contract prices $(\mathbf{p}, \boldsymbol{\pi})$, each agent $h \in H$ choses consumption, $\mathbf{c}$, asset holdings, $\mathbf{y}$, and contract sales/purchases, $\boldsymbol{\varphi}$, to maximize utility (Equation 5) subject to the budget set defined by

$$B^h(\mathbf{p}, \boldsymbol{\pi}) = \Big\{ (\mathbf{c}, \mathbf{y}, \boldsymbol{\varphi}) \in R_+^S \times R^{SK} \times \left(R^{J_s} \right)_{s \in S \setminus S_T} : \forall s$$
$$(c_s - e_s^h) + \mathbf{p}_s \cdot (\mathbf{y}_s - \mathbf{y}_{s^*} - \mathbf{a}_s^h) \leq$$
$$\leq \sum_{k \in K} d_s^k \left(y_{s^* k} + a_{sk}^h \right) + \sum_{j \in J_s} \varphi_j \pi_j - \sum_{k \in K} \sum_{j \in J_{s^*}^k} \varphi_j \min \left(b(j), p_{sk} + d_s^k \right);$$
$$\sum_{j \in J_s^k} \max \left(0, \varphi_j \right) \leq y_{sk}, \forall k \Big\}.$$

In each state s, expenditures on consumption minus endowments, plus the total expenditures on assets net of asset holdings carried over from the previous period and asset endowments, can be at most equal to the total asset deliveries plus the money borrowed selling contracts, minus the payments due at s from contracts sold in the past. Finally, those agents who borrow must hold the required collateral.

2.1.5. Collateral equilibrium.

A collateral equilibrium in this economy is a vector of financial asset prices and contract prices, consumption decisions, and financial decisions on assets and contract holdings $\left((\mathbf{p}, \boldsymbol{\pi}), (\mathbf{c}^h, \mathbf{y}^h, \boldsymbol{\varphi}^h)_{h \in H} \right) \in \left(R_+^K \times R_+^{J_s} \right)_{s \in S \setminus S_T} \times \left(R_+^S \times R_+^{SK} \times (R^{J_s})_{s \in S \setminus S_T} \right)^H$ such that

$$\sum_{h \in H} \left(c_s^h - e_s^h \right) = \sum_{h \in H} \sum_{k \in K} d_{s^*}^k \left(y_{s^* k}^h + a_{sk}^h \right), \forall s,$$

$$\sum_{h \in H} \left(y_s^h - y_{s^*}^h - a_s^h \right) = 0, \forall s,$$

$$\sum_{h \in H} \varphi_j^h = 0, \forall j \in J_s, \forall s,$$

$$(\mathbf{c}^h, \mathbf{y}^h, \boldsymbol{\varphi}^h) \in B^h(\mathbf{p}, \boldsymbol{\pi}), \forall h$$
$$(\mathbf{c}, \mathbf{y}, \boldsymbol{\varphi}) \in B^h(\mathbf{p}, \boldsymbol{\pi}) \Rightarrow U^h(\mathbf{c}) \leq U^h(\mathbf{c}^h), \forall h.$$

Markets for consumption, assets, and promises clear in equilibrium, and agents optimize their utility in their budget sets. Geanakoplos & Zame (1997) show that equilibrium in this model always exists.

2.2. A First Example

To fix ideas and motivate our main theoretical results, let us first consider a simple static example, similar to one in Geanakoplos (1997). Suppose $T = 1$, $S = \{0, U, D\}$, and suppose that there

is only one asset, Y, that pays dividends $d_U = 1$ and $d_D = 0.2$. Suppose the set of contracts $J = J_0 = \{1, 2, \ldots, 1{,}000\}$, where $b(j) = j/100$ for all j.

There are two types of agents $H = \{O, P\}$, with logarithmic utilities, who do not discount the future. Agents differ in their beliefs and wealth. Optimists assign a probability $\gamma_U^O = 0.9$ to the good state, whereas pessimists assign a probability of only $\gamma_U^P = 0.4$ to the same realization. Both agents are endowed with a single unit of the asset at the beginning, $a_0^h = 1, h = O, P,$ and are endowed with consumption goods: $e_0^O = e_D^O = 8.5$, $e_U^O = 10$, and $e_s^P = 100, s = 0, U, D$.

Table 1 describes the essentially unique equilibrium in this economy.[5] The price of the asset is $p = 0.708$. Optimists hold all the assets in the economy and use them all as collateral to borrow money from the pessimists. It turns out that the only contract traded in equilibrium is $j^* = 20$, $b(j^*) = d_D = 0.2$, which sells for the price $\pi_{j^*} = 0.199$. Optimists use two units of the assets as collateral to sell two units of the contract that promises to pay $b(j^*) = 0.2$, avoiding default in equilibrium. Thus, they borrow $2\pi_{j^*} = 0.398$. The resulting asset leverage is LTV $= (\pi_{j^*}/p) = (0.199/0.708) = 0.282$.

In equilibrium, all contracts are priced, even those that are not traded. For $b(j) \le 0.2$, we find that $\pi_j = b(j)/1.001$. One can borrow on these contracts at the same riskless rate of interest of 0.1%. For $b(j) > 0.2$, we see that $\pi_j = \big(0.4 \min\{b(j), 1\} + 0.6 \min\{b(j), 0.2\}\big)/1.001 < b(j)/1.001$. Because these contracts involve default, the associated interest rate $1 + r_j = b(j)/\pi_j$ is much higher than the riskless rate. For example, for $b(j) = 0.3$, we find that $\pi_j = 0.239$, and the interest rate is $r_j = 25.12\%$.

The prices of the contracts in the equilibrium described above correspond to the marginal utilities of the pessimists, so they are indifferent between lending or not. The optimists strictly prefer not to take any contract with $b(j) \ne 0.2$. For $b(j) > 0.2$, borrowing more by selling an infinitesimal amount of j instead of j^* means losing $0.9(b(j) - 0.2)/11.6$ infinitesimal utiles in state U and gaining only $(0.4/1.001)(b(j) - 0.2)/8.2$ infinitesimal utiles in state 0.

The asset, however, is priced according to the marginal utilities of optimists.[6] The key equations to calculate the equilibrium are thus[7]

$$\frac{1}{c_0^O}\left(p - \pi_{j^*}\right) = \gamma_U^O \frac{1}{c_U^O}(d_U - d_D) + \gamma_D^O \frac{1}{c_D^O}(d_D - d_D), \tag{6}$$

$$\frac{1}{c_0^P}\pi_{j^*} = \gamma_U^P \frac{1}{c_U^P}d_D + \gamma_D^P \frac{1}{c_D^P}d_D. \tag{7}$$

Equation 6 requires that the marginal utility to the optimists of the down payment for the asset is equal to their marginal utility of the asset dividends net of the j^* loan deliveries. The usual requirement, that the marginal utility of the asset price is equal to the marginal utility of the asset

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[5]This is in the sense that one can modify the prices of contracts that are not traded without disturbing agent maximization or market clearing (see the discussion in footnote 6).

[6]Thus, the equilibrium shown in **Table 1** has asset and contract prices that cannot be determined by state prices. However, this is not literally the unique equilibrium. One can modify the prices of contracts that are not traded without disturbing agent maximization or market clearing. Fostel & Geanakoplos (2013) show that in binomial trees with debt contracts and one financial asset (as in our example), there is always an equilibrium with unique state prices explaining the asset price and all the contract prices. In this example, the state price for U is 0.635 and that for D is 0.363.

[7]To solve for equilibrium, we guess the regime in **Table 1** in which the optimists hold all the assets and leverage to the maxmin: We set $y^O = 2$, $y^P = 0$, $\varphi_{j^*}^O = 2$, and $\varphi_{j^*}^P = -2$. These clearly satisfy market clearing. Equations 6 and 7, jointly with each agent budget set, determine the asset price, debt contract price, and individual consumptions. Finally, we check that the regime assumed is indeed a genuine equilibrium. To do this, we need to check that the following conditions hold for the equilibrium values calculated: $(1/c_0^O)p \ge \gamma_U^O(1/c_U^O)1 + \gamma_D^O(1/c_D^O)d_D$ and $(1/c_0^O)\pi_{j^*} \ge \gamma_U^O(1/c_U^O)d_D + \gamma_D^O(1/c_D^O)d_D$.

Table 1 Equilibrium static economy

States	$s = 0$	$s = U$	$s = D$
Prices and leverage			
p	0.708		
$b(j^*)$	0.2		
π_{j^*}	0.199		
r_{j^*}	0.1%		
LTV	0.282		
Asset Y			
Optimists	2		
Pessimists	0		
Debt contracts φ_j			
Optimists	2		
Pessimists	−2		
Consumption			
Optimists	8.2	11.6	8.5
Pessimists	100.3	100.4	100.4

Abbreviation: LTV, loan to value.

dividends, does not necessarily hold in a collateral equilibrium. Neither does the usual requirement that the marginal utility to the optimists of a dollar borrowed must equal their marginal utility of the deliveries they end up making on the loan. Sections 2.5–2.7 explain why these other conditions need not always hold in a collateral equilibrium. There are several important ideas coming out of this simple example that we discuss now and further formalize in Sections 2.3–2.7.

2.2.1. Endogenous leverage.   First, in equilibrium, there is not just one interest rate but a menu of interest rates depending on the promise per unit of collateral. A problem when calculating equilibrium is to know which contract is actively traded. Agents have access to a whole menu of contracts J_0, all of which are priced in equilibrium. But because collateral is scarce (there are only two units of collateral in the economy, and in an Arrow-Debreu equilibrium, promises would be much bigger), only a few contracts will be traded. As Geanakoplos (1997) explains, all contract types are not rationed equally; instead, most will be rationed to zero trade, and just a few, possibly just one type, will be actively traded in equilibrium. The example shows that only one contract is traded, the maxmin contract j^* satisfying $b(j^*) = d_D = 0.2$. This is the maximum amount optimists can promise while guaranteeing that they will not default in the future. One might have thought that optimists would be so eager to borrow money to buy the asset from pessimists (who they believe undervalue the asset) that they would want to promise more than 0.2 per asset, happily paying a default premium to get more money at time 0. According to the equilibrium, this is not the case.

Second, the reason optimists do not borrow more is that they are constrained from borrowing more at the going interest rate. The equilibrium interest rate is $r = 0.1\%$, and at that rate, optimists would be willing to borrow much more than 0.398, even if they were forced to deliver in full (out of

their future endowments) under punishment of death.[8] But no lender will be willing to lend them more money at 0.1% interest because there is no punishment. There is only collateral to enforce the delivery, and promises beyond 0.2 per asset will default in state D. The only way to borrow more, while using the same collateral, is to sell a contract j with $b(j) > 0.2$. But as we explain above, such contracts are priced by the market at much higher interest rates than 0.1%. Of course, the borrowers are also aware that they will default and not actually have to pay everything they owe in state D, and as a result, they are willing to pay a default premium. But this default premium is not enough to satisfy the lenders: The equilibrium implied interest rate on j with $b(j) > 0.2$ is still higher, so borrowers will not borrow more. The threat of default is so strong, it causes the lenders to constrain the borrowers. More precisely, the offered interest rate rises too fast as a function of $b(j)$ for the borrowers to be willing to take on more debt.

Third, leverage is not restricted a priori, but in equilibrium leverage is characterized by a simple formula. Each contract has a leverage associated with it. When only one contract is traded in equilibrium, this uniquely pins down the leverage in the economy. Moreover, because there is no default, the price of the only traded contract is given by $\pi_{j^*} = d_D/(1 + r_j^*)$. Leverage can then be characterized by the simple formula LTV $= \pi_{j^*}/p = (d_D/p)/(1 + r_{j^*})$. Thus, the LTV is given by the ratio between the worst-case rate of return on the asset and the riskless rate of interest.

2.2.2. Leverage raises asset prices.
The collateral equilibrium asset price $p = 0.708$ is much higher than its price in Arrow-Debreu equilibrium. With complete markets, the pessimists are so rich that they can insure the optimists without greatly disturbing their marginal utilities. These in turn will determine the Arrow prices. The Arrow prices are $p_U = 0.427$ and $p_D = 0.556$, which give an asset price of $p = p_U 1 + p_D 0.2 = 0.539$. Thus, leverage can dramatically raise asset prices above their efficient levels.

One might think that short-sale constraints would suffice to explain high asset prices. At $p = 0.708$, pessimists would like to short the asset but cannot. What would happen if we dropped leverage but still prohibited short selling? In the ensuing equilibrium, optimists would buy all of the asset; therefore, indeed their marginal utilities alone would determine the asset price. Nevertheless, the asset price would only be $p = 0.609$. One reason that the no-leverage price is much lower than the leverage price is that the optimists need to give up so much consumption at time 0 to buy the assets (as they cannot borrow) that their marginal utilities of the asset payoffs relative to the marginal utility of consumption at date 0 become low.

Additionally, the asset price in collateral equilibrium is higher than its marginal utility to every agent, even to the agents who buy it. In this example, the payoff value of the asset for the optimist is $\mathrm{PV}^O = \left(\sum_{s=U,D} \delta^O \gamma_s^O d_s du^O(c_s^O)/dc\right)/\left(du^O(c_0^O)/dc\right) = 0.655$. For the pessimists, the payoff value is much lower. Yet the price is $p = 0.708 > \mathrm{PV}^O = 0.655$. The reason the optimists are willing to pay more for the asset than its payoff value to them is that holding more of the asset enables them to borrow more money. This is what Fostel & Geanakoplos (2008) call collateral value.

Harrison & Kreps (1978) emphasize that short-sale constraints could raise the price of assets. Geanakoplos (2003) shows that leverage could raise the price of assets substantially more. Fostel & Geanakoplos (2012a, 2013) demonstrate that for one family of economies, the leverage price is always higher than the no-short-selling price, which is higher than the Arrow-Debreu price.[9]

---

[8]Even after borrowing 0.398, the marginal utility of one unit of consumption at time 0 is 1.37 times bigger than the expected marginal utility of consumption at time 1.

[9]None of these price rankings is universally true. For example, if the utilities are linear, then the collateral equilibrium price does not depend on the future endowments, but the Arrow-Debreu price does. Thus, by manipulating future endowments, one could make the Arrow-Debreu price higher or lower than the leverage price.

2.2.3. Collateral value and bubbles. Harrison & Kreps (1978) define a bubble as a situation in which an asset trades for a price that is above every agent's payoff value. They show that a bubble could emerge in equilibrium if there were at least three periods because the buyer in the first period could sell it in the second period to somebody who valued it more than he or she did from that point on. As the example demonstrates, the presence of collateral is enough to generate a bubble even in a two-period model. The buyer of an asset gets its payoff value as usual but also gets a positive collateral value from being able to borrow more as a result of holding it.

   Collateral value was missing in other early work on collateral, such as in Kiyotaki & Moore (1997), because in their model consumption effectively was driven to 0.[10] It seemed in their example as if collateral was undervalued because the marginal utility of its payoffs was greater than its equilibrium price, while the marginal utility of consumption was equal to its price. But when consumption is 0, this is a meaningless comparison. In their model (as always in a collateral equilibrium when the borrowing constraint is binding), a dollar's worth of collateral payoffs gives less marginal utility than a dollar can bring if it is spent optimally (which would be not on consumption but on the down payment for still more collateral). Measuring the marginal utility of a dollar properly, even in the Kiyotaki & Moore example, one finds that collateral is overvalued. Other papers develop the concept of collateral value. Garleanu & Pedersen (2011) define collateral value in a capital asset pricing model economy with agents with different risk aversion. Lagos (2010) generates liquidity premiums in a search and matching setting. Geanakoplos & Zame (2014) give a long discussion of collateral value and liquidity value. An early study of the subject in a partial equilibrium model is provided by Hindy (1994).

2.3. Absence of Default

In the example above, despite agents having access to a whole menu of contracts, we see that in equilibrium optimists borrow only through the maximum contract that prevents default. This is a general property of this class of models. The binomial no-default theorem states that in binomial economies with financial assets serving as collateral, every equilibrium is equivalent (in real allocations and prices) to another equilibrium in which there is no default. Thus, in binomial economies with financial assets, potential default has a dramatic effect on an equilibrium, but actual default does not.

> **Binomial no-default theorem:** Suppose that S is a binomial tree, that is, $S(s) = \{sU, sD\}$ for each $s \in S \backslash S_T$. Suppose that all assets are financial assets and that every contract is a one-period debt contract.
>
> Let $((\mathbf{p}, \boldsymbol{\pi}), (c^h, y^h, \boldsymbol{\varphi}^h)_{h \in H})$ be an equilibrium. Suppose that for any state $s \in S \backslash S_T$ and any asset $k \in K$, the maxmin contract $j^*(s, k)$ defined by $b(j^*(s, k)) = \min\{p_{sUk} + d_{sU}^k, \ p_{sDk} + d_{sD}^k\}$ is available to be traded [i.e., $j^*(s, k) \in J_s$]. Then we can construct another equilibrium $[(\mathbf{p}, \boldsymbol{\pi}), (c^h, \overline{y}^h, \overline{\boldsymbol{\varphi}}^h)_{h \in H}]$ with the same asset and contract prices and the same consumption choices, in which only the maxmin contracts are traded.
>
> For a proof, readers are referred to Fostel & Geanakoplos (2013).[11]

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[10]Farmers consume the bruised fruit in equilibrium, but what is crucial is that this bruised fruit is not marketable.

[11]The binomial no-default theorem is valid in a more general context than the one considered in this article. It is valid with arbitrary preferences and endowments, contingent and noncontingent promises, many assets, many consumption goods, multiple periods, and production.

According to the binomial no-default theorem, in searching for an equilibrium in our example of Section 2.2, we never need to look beyond the maxmin promise $b(j^*) = 0.2$, for which there is no default. The promise per unit of collateral is unambiguously determined simply by the payoffs of the underlying collateral, independent of preferences or other fundamentals of the economy. Agents will promise as much as they can, while assuring their lenders that the collateral is enough to guarantee delivery.[12]

The theorem provides a hard limit on borrowing. Therefore, it shows that there must be a robust class of economies in which agents would like to borrow more at going riskless interest rates but cannot, even when their future endowments are more than enough to cover their debts.

The hard limit on borrowing is caused by the specter of default, despite the absence of default in equilibrium. If the asset payoff in the down state were to deteriorate, creating a clearer and more present danger of default, lenders would tighten credit.

The hard limit is endogenous. Lenders willingly offer contracts $j > j^*$ on which there would be default, but they charge such high interest rates that borrowers never choose them. One might have thought that the volume of trade in loans that default and loans that do not default could be the same. The defaulting loans would simply trade at higher interest rates, reflecting a default premium. However, the theorem shows that this is not the case.

Binomial economies and their Brownian motion limit are special cases. But they are extensively used in finance. They are the simplest economies in which one can begin to see the effect of uncertainty on credit markets. With multiple states, default could emerge in equilibrium. Moreover, some borrowers might use collateral to take loans that would default, while other borrowers might use the same collateral to take out loans in which delivery is fully guaranteed. Thus, the no-default and maxmin uniqueness properties do not extend beyond binomial economies. However, even in more general economies, borrowers would still be constrained, in the sense that they would not be able to borrow more at the same interest rates (unless they put up more collateral). The binomial case is the simplest and starkest setting in which one can clearly connect the risk of default and the tightness of credit markets.[13]

Geanakoplos (2003, 2010) and Fostel & Geanakoplos (2008, 2012a,b, 2013) work with binomial models of collateral equilibrium with financial assets, showing in their various special cases that, as the binomial no-default theorem implies, only the VaR = 0 contract is traded in equilibrium. Many papers have given examples in which the no-default theorem does not hold. Geanakoplos (1997) gives a binomial example with a nonfinancial asset (a house, from which agents derive utility), in which equilibrium leverage is high enough that there is default. Geanakoplos (2003) provides an example with a continuum of risk-neutral investors with different priors and three states of nature in which the only contract traded in equilibrium involves default. Simsek (2013) gives an example with two types of investors and a continuum of states of nature with equilibrium default. Araujo et al. (2012) provide a two-period example of an asset used as collateral in two different actively traded contracts when agents have utility over the asset. Fostel & Geanakoplos (2012b) present an example with three periods and multiple contracts traded in equilibrium.

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[12]The binomial no-default theorem does not say that equilibrium is unique, only that each equilibrium can be replaced by another equivalent equilibrium in which there is no default. However, as Fostel & Geanakoplos (2013) also show, among all equivalent equilibria, the maxmin equilibria (which never involve default) use the least amount of collateral. These collateral-minimizing equilibria would naturally be selected if there were the slightest transactions cost in using collateral or handling default.

[13]Even in binomial economies, we would observe default in equilibrium if we were to consider nonfinancial assets as collateral. But it would still be the case that borrowers are constrained.

2.4. Endogenous Leverage

In our static example above, we see that leverage is characterized by a very simple formula. As the following result shows, this is a general characterization for leverage in the class of binomial economies with financial assets.

> **Binomial leverage theorem:** Under the assumptions of the binomial no-default theorem, equilibrium leverage can always be taken to be
>
> $$\text{LTV}_s^k = \frac{d_{sD}^k / p_{sk}}{1 + r_s} = \frac{\text{worst-case rate of return}}{\text{riskless rate of interest}}.$$
>
> For a proof, readers are referred to Fostel & Geanakoplos (2013).

Equilibrium leverage depends on current and future asset prices, and the riskless rate of interest, but is otherwise independent of the utilities or the endowments of the agents. The theorem shows that in binomial models, it makes sense to use the VaR = 0 rule, assumed by many other papers in the literature.

Although simple and easy to calculate, the binomial leverage formula provides interesting insights. First, it explains why changes in the bad tail can have such a big effect on equilibrium, even if they hardly change expected payoffs: They change leverage. The theorem suggests that one reason leverage might have plummeted from 2006 to 2009 is because the worst-case return that lenders imagined got much worse. Second, the formula also explains why high leverage histori-cally correlates with low interest rates (even with rational agents who do not blindly chase yield). Finally, it explains which assets are more leveraged: The asset whose future return has the least bad downside will be leveraged the most.

Collateralized loans always fall into two categories. In the first category, borrowers are not designating all the assets they hold as collateral for their loans. In this case, they would not want to borrow any more at the going interest rates, even if they did not need to put up collateral (but were still required, by threat of punishment, to deliver the same payoffs they would had they put up the collateral). Their demand for loans is then explained by conventional textbook considerations of risk and return. In the second category, borrowers are posting all of their assets as collateral. In this case of scarce collateral, they are constrained by the specter of default: To borrow more, they may be forced to pay sharply higher interest rates. In binomial models with financial assets, the equilibrium LTV can be taken to be the same easy-to-compute number, no matter which category the loan is in (i.e., whether it is demand or supply determined).

The distinction between plentiful and scarce capital all supporting loans at the same LTV suggests that it is useful to keep track of a second kind of leverage, which we call diluted leverage, in which the denominator includes assets not used as collateral:[14]

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[14]Consider the following example: If the asset is worth $100 and its worst-case payoff determines a debt capacity of $80, then in equilibrium we can assume that all debt loans written against this asset will have the LTV equal to 80%. If an agent who owns the asset only wants to borrow $40, then he or she could just as well put up only half of the asset as collateral, as that would ensure there would be no default. The LTV would then again be $40/$50 or 80%. Hence, it is useful to consider diluted loan to value (DLTV), namely the ratio of the loan amount to the total value of the asset, even if some of the asset is not used as collateral. The DLTV in this example is 40% because the denominator includes the $50 of the asset that was not used as collateral.

Changes may still occur before final publication online and in print

$$\text{DLTV}_s^k = \frac{\sum_h \sum_{j \in J_s^k} \max\left(0, \varphi_j^b\right) \pi_j}{\sum_h y_{sk}^b p_{sk}} \leq \text{LTV}_s^k. \tag{8}$$

Similarly, one can define diluted investor leverage

$$\text{DLTV}_s^b = \frac{\sum_k \sum_{j \in J_s^k} \max\left(0, \varphi_j^b\right) \pi_j}{\sum_k y_{sk}^b p_{sk}} \leq \text{LTV}_s^b. \tag{9}$$

In binomial economies, LTV must always be exactly the worst-case return over the riskless rate of interest, but DLTV can be a smaller number. It is often said that leverage should be related to volatility: the lower the volatility, the higher the leverage. It turns out that this is the case in binomial economies with only one financial asset.

> **Binomial leverage-volatility theorem:** Under the assumptions of the binomial no-default theorem, for each state $s \in S \backslash S_T$, and each asset $k \in K$, there are risk-neutral pricing probabilities $\alpha = p_U(s, k)$ and $\beta = 1 - \alpha = p_D(s, k)$ such that the equilibrium price p_{sk} and equilibrium margin $m_s^k = 1 - \text{LTV}_s^k$ can be taken equal to
>
> $$p_{sk} = \frac{\alpha\left(p_{sUk} + d_{sU}^k\right) + \beta\left(\left(p_{sDk} + d_{sD}^k\right)\right)}{1 + r_s},$$
>
> $$m_s^k = \sqrt{\frac{\alpha}{\beta}} \frac{\text{Vol}_{\alpha,\beta}(k)}{(1 + r_s)p_{sk}},$$
>
> where $\text{Vol}_{\alpha,\beta}(k) = \sqrt{\alpha\beta}(p_{sUk} + d_{sU}^k - p_{sDk} - d_{sD}^k)$.
>
> For a proof, readers are referred to Fostel & Geanakoplos (2013).

The theorem states that the equilibrium margin on an asset is proportional to the volatility of a dollar's worth of the asset. The trouble with this theorem is that the risk-neutral pricing probabilities α and β depend on the asset k. If there were two different assets k and k' coexisting in the same economy, we might need different risk-neutral probabilities to price k and k'. Ranking the leverage of assets by the volatility of their payoffs would fail if we tried to measure the various volatilities with respect to the same probabilities.

2.5. Liquidity Value and Credit

In our example in Section 2.2, we see that agents were not able to borrow as much as they would like at the going interest rate. They were willing to pay a much higher interest to get their hands on extra money today. We now introduce concepts that help us precisely quantify the tightness of credit markets. Let us begin by defining the marginal utility to agent h associated with trading contract j at state s, assuming that consumption is positive at s, sU, and sD.[15]

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[15]If consumption $c_s = 0$, then the definition of the payoff value must be modified by replacing the marginal utility of s consumption by the marginal utility of money in state s.

Definition 1: The payoff value of contract j to agent h at state s is

$$PV_{sj}^{h} = \frac{\sum_{\sigma \in \{U,D\}} \delta_h \gamma_{s\sigma}^h \min\left\{ b(j), p_{s\sigma k(j)} + d_{s\sigma}^{k(j)} \right\} du^h\left(c_{s\sigma}^h\right)/dc}{du^h\left(c_s^h\right)/dc}. \tag{10}$$

Definition 2: The liquidity value LV_{sj}^h associated with contract j to agent h at s is

$$LV_{sj}^h = \pi_j - PV_{sj}^h. \tag{11}$$

The liquidity value represents the surplus a borrower can gain by borrowing money today selling a contract j backed by collateral k.

2.6. Liquidity Wedge and Discount Rate

The liquidity value gives an expression of how much less borrowers would take and still be willing to sell the same promise. Another way of saying that they find the loan beneficial on the margin is by defining the following.

Definition 3: The liquidity wedge ω_{sj}^h associated with contract j for agent h at state s is

$$1 + \omega_{sj}^h = \frac{\pi_j}{PV_{sj}^h}. \tag{12}$$

In the case in which contract j fully delivers, ω_{sj}^h defines the extra interest potential borrowers would be willing to pay above the going riskless interest rate if they could borrow an additional penny and were committed (under penalty of death) to fully deliver. This extra interest is called the liquidity wedge; it gives a measure of how tight the contract j credit market is. We have seen that in binomial economies, agents only take out riskless loans. It is obvious that there cannot be two riskless loans actively trading for different interest rates, for that would mean that the lender who got the lower interest rate had made a mistake. Hence, we can unambiguously define the state s liquidity wedge ω_s^h for any agent h who actively borrows there.

The liquidity wedge can be given another very important interpretation as shown in the following theorem.

Discount theorem: Define the risk-adjusted probabilities for agent h in state s by

$$\mu_{sU}^h = \frac{\gamma_{sU}^h du^h\left(c_{sU}^h\right)/dc}{\gamma_{sU}^h du^h\left(c_{sU}^h\right)/dc + \gamma_{sD}^h du^h\left(c_{sD}^h\right)/dc},$$

$$\mu_{sD}^h = \frac{\gamma_{sD}^h du^h\left(c_{sD}^h\right)/dc}{\gamma_{sU}^h du^h\left(c_{sU}^h\right)/dc + \gamma_{sD}^h du^h\left(c_{sD}^h\right)/dc} = 1 - \mu_{sU}^h.$$

If agent h is taking out a riskless loan in state s, then his or her payoff value in state s for a tiny share of cash flows consisting of consumption goods $x = (x_{sU}, x_{sD})$ is given by

$$PV^h(x) = \frac{\mu_{sU}^h x_{sU} + \mu_{sD}^h x_{sD}}{(1 + r_s)(1 + \omega_s^h)}.$$

For a proof, readers are referred to Fostel & Geanakoplos (2008).

This result is important because it shows that in evaluating assets that he or she might purchase, an agent who is borrowing constrained will discount the cash flows by a spread above the riskless rate; this spread is the same for all cash flows. As the agent becomes more liquidity constrained, in the sense of having a higher liquidity wedge, his or her willingness to pay for all assets will decline. The only exception might be for some assets that can serve as such good collateral that they bring an additional collateral value of enabling their owner to issue more loans.

2.7. Collateral Value and Asset Pricing

An asset's price reflects not only its future returns but also its ability to be used as collateral to borrow money. Consider a collateral equilibrium in which an agent h holds an asset k at state $s \in S$, and suppose h consumes a positive amount in each state. As seen in the example of Section 2.2, when the asset can be used as collateral and the collateral constraint is binding, the asset price can exceed the agent's asset valuation given by the payoff value defined as follows.

Definition 4: The payoff value of asset k to agent h at state s is

$$\mathrm{PV}_{sk}^h \equiv \frac{\sum_{\sigma \in \{U,D\}} \delta_h \gamma_{s\sigma}^h \left(p_{s\sigma k} + d_{s\sigma}^k \right) du^h \left(c_{s\sigma}^h \right) / dc}{du^h \left(c_s^h \right) / dc}. \tag{13}$$

Definition 5: The collateral value of asset k in state s to agent i is

$$\mathrm{CV}_{sk}^h \equiv p_{sk} - \mathrm{PV}_{sk}^h. \tag{14}$$

The collateral value stems from the added benefit of enabling borrowing that some durable assets provide. Collateral values distort pricing and typically destroy the efficient markets hypothesis, which in one of its forms asserts that there are risk-adjusted state probabilities that can be used to price all assets. Some assets may bring lower returns to investment, even accounting for the riskiness of the returns, because their prices are inflated by their collateral values.

Collateral value = liquidity value theorem: Suppose that $y_{sk}^h > 0$ and $\varphi_j^h > 0$ for some agent h and some $j \in J_s^k$. Then, in equilibrium the following holds:

$$\mathrm{LV}_{sj}^h = \mathrm{CV}_{sk}^h.$$

The liquidity value associated with any contract j that is actually issued using asset k as collateral equals the collateral value of the asset.

For a proof, readers are referred to Fostel & Geanakoplos (2008) and Geanakoplos & Zame (2014).

The collateral value is the additional cost an agent is willing to pay above the payoff value because he or she can use the asset as collateral. The liquidity value is the benefit of borrowing through a contract that uses the asset as collateral. In equilibrium, these two are the same.

No agent will overpay for the collateral unless he or she can gain at least as much liquidity value. If the liquidity value were more, then the agent would not be content and would buy more collateral to issue still more loans. In collateral equilibrium, agents are never barred from borrowing more; they can always put up more collateral. They act as if they were constrained by choosing not to borrow

more, even though there is a positive liquidity value to borrowing, because the collateral is too expensive.

The equality demonstrated in the theorem is the key equation in computing a collateral equilibrium. It is equivalent to Equation 6, which asserts that the difference in payoff value between the collateral and the loan has to be equal to the down payment on the collateral.

2.8. Liquidity and Endogenous Contracts

Because one collateral cannot back many competing loans, the borrower will always select the loan that gives the highest liquidity value among all loans with the same collateral. This leads to a theory of endogenous contracts in collateral equilibrium models and, in particular, to a theory of endogenous leverage, as seen in Sections 2.3 and 2.4. From Definitions 2 and 3, it is clear that the liquidity value and liquidity wedge satisfy the following for any contract j:

$$\mathrm{LV}^b_{sj} = \mathrm{PV}^b_{sj}\omega^b_{sj}. \tag{15}$$

All loans that deliver for sure will have the same liquidity wedge. If this wedge is positive, the borrower will naturally choose the biggest loan, as that has the highest payoff value and therefore the highest liquidity value. That explains why in binomial economies, the borrower always prefers the maxmin contract to all contracts that promise strictly less. Borrowers could also gain a surplus from contracts that promise more and default. But the lenders require a sharply higher interest, so the liquidity wedge declines rapidly as loans default more. As a result, the borrowers voluntarily choose to trade only the maxmin contract.[16]

3. LEVERAGE CYCLE

We now study the dynamic implications of the results presented in Section 2. We see how, in a dynamic context, leverage and asset prices engage in a positive feedback, rising together then falling together, to create something we call the leverage cycle.

We extend the static example of Section 2 to a three-period economy, so $T = 2$, and $S = \{0, U, D, UU, UD, DU, DD\}$. There is one financial asset Y, which pays dividends only in the final period. We follow Geanakoplos's (2003) model, which has a continuum of risk-neutral agents, as adapted by Fostel & Geanakoplos (2008) to two risk-averse agents.

The tree of asset payoffs has the property that good news reduces uncertainty about the payoff value and that bad news increases uncertainty about the payoff value of the asset. We assume, as shown in **Figure 2**, that after good news at $s = U$, the asset payoff is equal to $d_{UU} = d_{UD} = 1$ with certainty. However, after bad news at $s = D$, the future payoff volatility increases. We assume that $d_{DU} = 1$ and $d_{DD} = 0.2$. The coincidence of bad news and increased volatility is the hallmark of the leverage cycle. We have seen that volatility tends to reduce leverage. Thus, the bad news in the leverage cycle will reduce expected payoffs at the same time it reduces leverage (for a more general treatment of volatility and the leverage cycle, see Fostel & Geanakoplos 2012b).

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[16]It is quite possible that a contract has a very high liquidity wedge associated with it; therefore, it might be very useful to introduce it into an economy in which agents could be counted on to deliver without posting collateral, but it is not chosen in a collateral equilibrium because it is small and therefore has a low payoff value and thus a low liquidity value. Such a promise might be useful in a GEI (general equilibrium with incomplete markets) economy but not in a collateral economy because it uses up too much collateral.
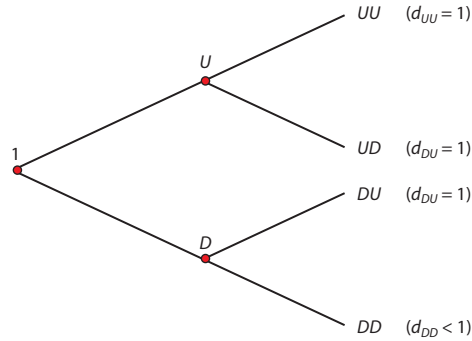
Figure 2

Asset Y payoffs.

As before, there are two types of agents $H = \{O, P\}$ with logarithmic utilities who do not discount the future. Agents differ in their beliefs and wealth. Optimists assign a probability $\gamma_{sU}^O = 0.9$ of moving up from any state $s \in S \backslash S_T$, whereas pessimists assign a probability for moving up of only $\gamma_{sU}^P = 0.4$ for all $s \in S \backslash S_T$. Both agents are endowed with a single unit of the asset at the beginning, $a_0^h = 1$, $h = O, P$, and an endowment of the consumption good in each state as follows: $e_0^O = e_D^O = 8.5$, $e_s^O = 10$, $s \neq 0, D$, and $e_s^P = 100, \forall s$.

Table 2 describes the essentially unique equilibrium in this economy.[17] By the no-default theorem, we know that the only contract traded in each node is the maxmin contract that prevents default. Because after good news there is no remaining uncertainty, the equilibrium decisions at that node are simple: There is no borrowing (as debt and the asset are perfect substitutes), and agents just trade the asset against the consumption good. At the initial node 0 and after bad news D, the situation is more subtle.

The equilibrium portfolios at 0 and D are of the same type as in the static example. Optimists hold all the assets in the economy and use them as collateral to borrow money from the pessimists. They buy the asset on margin, selling the maxmin contract at each node: At 0, they promise the price of the asset p_D after bad news, and at D, they promise 0.2 per each unit of the asset.

3.1. Ebullient Times

Collateral is usually scarce; borrowing is usually constrained. But when volatility is low, as at $s = 0$, the existing scarce collateral can support large amounts of borrowing to buy assets that are acceptable collateral. If there is sufficient heterogeneity among agents in their enthusiasm for holding the asset, and short selling is not allowed, a bubble can emerge in which the prices of the assets that can be used as collateral rise to levels far above their Arrow-Debreu Pareto-efficient levels, even though all agents are rational. In this example, leverage at time 0 is almost 4 to 1 (LTV = 0.73), and the asset price at time 0 is 0.91. In an Arrow-Debreu equilibrium, the asset price would only be 0.71. The price is so high in the leverage equilibrium because the pessimists have no way to express their opinions about the asset except by selling. The optimists not only can buy out of their endowments, they can also borrow and buy more, leveraging their opinion. On top of all that, the

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[17]To calculate the equilibrium, we use the same logic as in Equations 6 and 7 for each node. Detailed equations and programs for all the examples in the review are available upon request.

optimists are willing to pay a collateral value of 0.06 above and beyond the asset payoff value of 0.85 to them, because holding it enables them to borrow more money.

The combination of high prices and low volatility creates an illusion of prosperity. But in fact the seeds of collapse are growing as the assets get increasingly concentrated in the hands of the most enthusiastic and leveraged buyers. When bad news that creates more uncertainty occurs, the bubble can burst.

3.2. The Crash

Leverage cycle crashes always occur because of a coincidence of three factors. The bad news itself lowers the prices. But it also drastically reduces the wealth of the leveraged buyers, who were leveraged the most precisely because they are the most optimistic buyers. Thus, the purchasing power of the most willing buyers is reduced. And most importantly, if the bad news also creates more uncertainty, then credit markets tighten and leverage will be reduced, just when the optimists would like to borrow more, making it much harder for the optimists and any potential new buyers to find funding.

The price of the asset in our example goes down from 0.91 at 0 to 0.67 at D after bad news, a drop of 24 points. At both 0 and D, the optimists are the only agents holding the asset, and in their view, the expected payoff of the asset drops only 7 points, from 0.99 to 0.92, after the bad news.

Table 2 Equilibrium leverage cycle

States	$s = 0$	$s = U$	$s = D$	$s = UU$	$s = UD$	$s = DU$	$s = DD$
Prices, leverage, and liquidity wedge							
p	0.909	0.982	0.670				
j^*	0.670		0.2				
π_{j^*}	0.664		0.201				
LTV	0.730		0.299				
PV	0.850	0.98	0.602				
CV	0.059	0	0.068				
ω	0.097	0	0.518				
Asset holdings							
Optimists	2	0.408	2				
Pessimists	0	1.591	0				
Debt contract							
Optimists	2		2				
Pessimists	−2		−2				
Consumption							
Optimists	8.92	10.22	7.56	10.41	10.41	11.6	10
Pessimists	99.58	99.78	100.94	101.59	101.59	100.4	100.4

Abbreviations: CV, collateral value; LTV, loan to value; PV, payoff value.
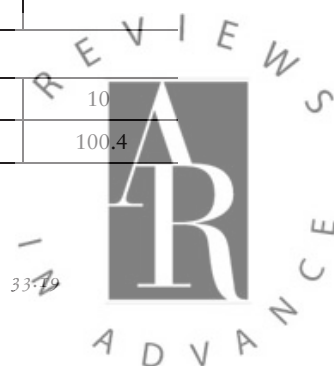
So there is something much more important than bad news that explains the drop in asset price. This is the downward path of the leverage cycle.

First, notice that the optimists, although still buying all the asset in the economy, lose wealth after bad news. At 0, they started with 8.5 units of the consumption good and half the assets. To maintain high consumption and to buy up the rest of the assets, which they regard as a good investment, they become so leveraged at 0 that they owe the value of all their assets at D; after paying, they are left with only their initial endowment of 8.5 consumption goods. So they get poorer at D and are forced to consume less if they want to repurchase all their assets. Second, the higher volatility at D reduces the amount they can leverage. Leverage plummets from 4 at 0 to 1.4 at D (equivalently, the LTV goes from 0.73 to 0.29). Optimists are forced to drastically scale back their consumption at D if they want to continue holding all the assets. In fact, they do want to continue because they regard their opportunity at D as even greater than that at 0. Indeed, the disagreement between optimists and pessimists over the value of the assets is higher at D than at 0. Curiously, optimists are able to borrow the least just when they feel the greatest need. As a result of their decreased consumption and their perception of a greater opportunity, their liquidity wedge, which is a measure of how much they are willing to pay above the riskless interest rate, increases dramatically, from 0.1 to 0.52. By the discount theorem, they then discount all future cash flows at a much higher rate than the riskless rate, and it is this extra discounting of the future that reduces the value of the assets so much more than the bad news. On account of the bigger discount, the payoff value of the assets sinks all the way to 0.60. Of course, there is still a collateral value of 0.07. But despite the high liquidity wedge, the collateral value of the assets is limited by the small amount of borrowing they support.

In summary, it is the combination of bad news, loss of current wealth (liquidity scarcity), and lower leverage that makes the crash in prices really dramatic. This evolution from low volatility and rising leverage and asset prices to high volatility and declining leverage and asset prices is the leverage cycle.

3.3. Margin Calls

The most visible sign of the crash is the margin call. After the bad news at D starts to reduce asset prices, optimists who want to roll over their loans need to put up more money to maintain the same LTV on their loans. They could do that either by selling assets or by reducing their consumption. In our example here, they choose to reduce their consumption. They then effectively get a second margin call because the new LTV is much lower than before, forcing them to reduce consumption further. The reduction in consumption increases the rate at which they discount future cash flows, and it is this more than the bad news that causes asset prices to crash. In his original model of the leverage cycle, Geanakoplos (2003) develops an alternative model of the leverage cycle in which the initial endowment of consumption goods of optimists is lower at D than at 0. When the margin call comes, they are too poor to hold the assets by cutting down on consumption and are forced to sell instead. The new buyers are less enthusiastic or optimistic about the assets than the original optimists, so the price crashes because the marginal buyer is a different and more pessimistic agent. The mechanism is analogous, whether the loss in value comes from the same agents discounting more or from new agents who value the assets less.

Brunnermeier & Pedersen (2009) provide a theory of margin calls, which they call margin spirals. Margins in their theory are exogenously set by the VaR = 0 rule. Margin calls arise in a context of multiple equilibria.

3.4. Maturity Mismatch

If the optimists had borrowed for two periods instead of for one, they would not be forced to reduce their consumption (or to sell) at D. One might have thought that in order to reduce this margin call risk at D, optimists would prefer to take out long-term debt instead of short-term debt at 0. Geanakoplos (2010) examines this question in a similar model and observes that even if they were given the choice of long-term debt, optimists would choose the short-term debt. In our current model, all debts are, by assumption, for one period. We could augment the current model by allowing noncontingent two-period debt as well as the short-term debt. If long-term debt could not be retraded in the middle periods, then the binomial no-default theorem could be immediately extended to long-term debt when the collateral only takes on two values across all the states of nature at which the bond payments come due. In this example, the collateral is worth either 1 or 0.2 across the four terminal states at time 2. Hence, we could conclude that among all long-term debt contracts, only the debt contract collateralized by one unit of the asset and promising 0.2 units of consumption in every terminal state might be traded in equilibrium. But the optimists would not want to borrow on that contract, as they could raise 0.67 instead of 0.199 by borrowing on the one-period contract and risking the unlikely (from their point of view) margin call at D.

3.5. Crisis Economy Versus Anxious Economy

When the crash comes at D, the optimists still feel things will turn around and think on average the asset will pay 0.92 in the end. Buying at D is an opportunity for them, as the asset has gone down very little in the expected payoff but has a much lower price. Fostel & Geanakoplos (2008) distinguish between the case in which optimists are forced to sell at D, which they call the crisis economy, and the case in which optimists have enough liquid wealth at D to maintain their assets and perhaps buy new ones, which they call the anxious economy. In the current example, the optimists are not forced to sell, but they do not buy more either. It is thus on the borderline between a crisis economy and an anxious economy.

3.6. Volatility

The signature of the leverage cycle is rising asset prices in tandem with rising leverage, followed by falling asset prices and leverage. But the underlying cause of the change in leverage is a change in volatility or, more generally, in some kind of bad tail uncertainty. In our example, the volatility of the asset's value is 0.126 at time 0, when leverage is almost 4, and increases to 0.394 at D, when leverage plummets to 1.4. The sharp increase in volatility mostly results from a technology shock. In the standard real business cycle literature, there are technology shocks that increase or decrease productivity, but there is not much attention paid to shocks that increase volatility. Leverage can also rise for endogenous reasons. After the optimists lose income at D, their expenditure on assets becomes much more sensitive to their wealth.

Many recent papers assume a link between leverage and volatility (see, e.g., Adrian & Boyarchenko 2012, Brunnermeier & Pedersen 2009, Thurner et al. 2012). Geanakoplos (2003, 2010) and Fostel & Geanakoplos (2008, 2012b) derive this link from first principles, as special cases of the binomial leverage theorem. Brunnermeier & Sannikov (2014) also derive leverage endogenously from first principles, but it is determined not by collateral capacities but by agents' risk aversion; it is a demand-determined leverage that would be the same without collateral requirements. The time series movements of the LTV come from movements in volatility because the added uncertainty makes

borrowers more scared of investing, rather than from a reduction of the debt capacity of the collateral as lenders are more scared to lend.

3.7. Smoothing the Leverage Cycle

Asset prices are much too high at 0 (compared to Arrow-Debreu first-best prices) and then crash at D, rising and falling in tandem with leverage. If we added investment and production of the asset into the model, we would find overproduction at 0 and then a dramatic drop in production at D. Macroeconomic stability policy has concentrated almost entirely on regulating interest rates. But interest rates over the cycle in the leverage cycle example barely move. The leverage cycle suggests that it might be more effective to stabilize leverage than to stabilize interest rates.

Optimists have a higher marginal propensity to buy the asset at 0 and D than do pessimists because they are more enthusiastic about the asset. Thus, regulating leverage to lower levels at 0 will not only lower the asset prices at 0, but will also raise the asset price at D because it will leave optimists less in debt. This will smooth the leverage cycle and move prices closer to Arrow-Debreu levels. In a slightly more complicated model, it will lead to Pareto improvements. It will not, however, lead to a Pareto improvement in this example, for an instructive reason.

In collateral equilibrium, borrowers are constrained from borrowing as much as they like, but lenders are not. If an increase in borrowing and lending could be arranged, it could make both borrowers and lenders better off, assuming that borrowers could be coerced into delivering fully out of their future endowments. Forcing a small reduction in credit is positively harmful to borrowers and has little effect on lenders, assuming that future prices do not change. This probably explains why government policy has been almost exclusively geared to expanding credit rather than reigning it in.

But in collateral equilibrium, insurance markets are often missing, as in the leverage cycle example. Curtailing credit will lead to price changes in the future, which have redistributive consequences that may be beneficial. Geanakoplos & Polemarchakis (1986) prove that when insurance markets are missing, there is almost always an intervention in financial markets at 0 that will induce future price changes that are Pareto improving. But when there is a positive liquidity wedge, the future Pareto improvement that might come from curtailing leverage must overcome the immediate effect of limiting an already constrained credit market.

In the leverage cycle example, optimists sell assets at U. But optimists and pessimists have identical utilities at U because there is no remaining uncertainty, and both have discount rates of 1. Thus, curtailing leverage at 0 does not affect prices at U. Curtailing leverage at 0 does raise the price of assets at D, but there is no trade in assets at D, as the optimists buy them all at 0 and retain them all at D. Thus, the increase in asset prices at D does not redistribute wealth and has a negligible effect on welfare.

We are thus led to consider a modified leverage cycle example in which pessimists have a discount rate of 0.95 and in which they are endowed with an additional 1.5 assets at both U and D, but that is otherwise the same as the leverage cycle example. The equilibrium is described in **Table 3**, as is the equilibrium after leverage is regulated to a smaller level at 0. In the modified leverage cycle example, curtailing leverage at 0 not only raises the price of assets at D, but also raises the price of assets at U because now optimists are more patient than pessimists and so will invest more of their extra money at U into assets than pessimists withdraw when they receive smaller debt payments. Because optimists are selling the asset at U, this price rise helps optimists and hurts pessimists. Moreover, because optimists care more about U than pessimists do, this increases the sum of utilities (normalized so that the marginal utility of consumption at 0 is 1 for all agents). At D, the

Table 3 Smoothing the leverage cycle

	Unrestricted leverage	Restricted leverage, $j = 0.58$	Leverage transfer, $j = 0.58$, $t = 0.0004$
Price at $s = 0$	0.820423	0.819622	0.819613
Price at $s = U$	0.925092	0.925097	0.925097
Price at $s = D$	0.590795	0.591873	0.591873
Utility optimists	60.0274	60.0279	60.0275
Utility pessimists	1,311.6860	1,311.6858	1,311.6862

optimists are buying the extra endowment of assets, so the price rise hurts optimists and helps the pessimistic sellers. But pessimists care more about D than do optimists, so the price change again raises total utility.

The increase in future total utility is more than the loss in total utility from curtailing the already rationed borrowing. But curtailing leverage has one more effect. It lowers the price of assets at 0, thereby helping optimists and hurting pessimists. To make all agents better off, the policy intervention should reduce leverage and transfer some consumption at time 0 from optimists to pessimists. Both these objectives could be achieved by taxing borrowing and then redistributing the revenue to all agents (and not back to those who paid the tax). **Table 3** shows that such an intervention is indeed Pareto improving.

The most important benefit from curtailing leverage is not captured by the modified leverage example because there is no default in binomial economies with financial assets. Geanakoplos & Kubler (2013) construct a multistate example with common beliefs in which there is heterogeneity because optimists get utility from housing. They are thus led to borrow so much on their mortgages that some of them will default in some of the states. Curtailing leverage has the extra benefit that by raising the future price of houses, it reduces default, as whether homeowners default depends on how far underwater they are. Although lenders rationally anticipate that by curtailing the loan, they can reduce the chances of their own borrower defaulting, they do not take into account that by lending less they can help increase future housing prices and thus reduce other borrowers' chances of defaulting. If defaulting homeowners neglect repairs on their houses, curtailing leverage can lead to Pareto improvements.

3.8. Agent Heterogeneity

The leverage cycle relies crucially on agent heterogeneity. In the example, heterogeneity was created by differences in beliefs. But there are many other sources of heterogeneity. Some agents are more risk tolerant than others. Some agents can use assets more productively than others. Some households like living in houses more than others. And some agents need assets to hedge more than others. It is very important to understand that the connection between leverage and asset prices does not rely on differences in beliefs.

To see this, consider a variant of our leverage cycle example in which agents have the same log utilities and identical beliefs so that $\gamma_{sU}^O = \gamma_{sU}^P = 0.5$ for all $s \in S \setminus S_T$. Endowments of the consumption good for the O group are $e_0^O = 8.5$, $e_U^O = 5.5$, $e_D^O = 38.8$, $e_{UU}^O = e_{UD}^O = 5.4$, $e_{DU}^O = 30.6$, and $e_{DD}^O = 250$ and for the P group are $e_0^P = 100$, $e_U^P = 125$, $e_D^P = 83.2$, $e_{UU}^P = e_{UD}^P = 125.4$, $e_{DU}^P = 104.2$, and $e_{DD}^P = 69.3$. For the O group, the asset is a natural hedge to their endowments; for the P group, the asset is not so useful. Starting with the same endowments of the

asset as in our leverage cycle example, equilibrium asset prices and portfolio trades are identical to those in the leverage cycle example displayed in **Table 2**.[18]

3.9. Lessons from the Leverage Cycle

The lessons from the leverage cycle are as follows. First, increasing leverage on a broad scale can increase asset prices. Second, leverage is endogenous and fluctuates with the fear of default. Third, leverage is therefore related to the degree of uncertainty or volatility of asset markets. Fourth, the scarcity of collateral creates a collateral value that can lead to bubbles in which some asset prices are far above their efficient levels. Fifth, the booms and busts of the leverage cycle can be smoothed best not by controlling interest rates, but by regulating leverage. Sixth, the amplitude of the cycle depends on the heterogeneity of the valuations of the investors.

3.10. Credit Cycle Versus Leverage Cycle

Our final observation is that a leverage cycle is not the same as a credit cycle. A leverage cycle is a feedback between asset prices and leverage, whereas a credit cycle is a feedback between asset prices and borrowing. If the LTV is fixed at a constant, then borrowing and asset prices rise and fall together. But leverage is unchanged. Of course, a leverage cycle always produces a credit cycle. But the opposite is not true. Classical macroeconomic models of financial frictions such as Kiyotaki & Moore (1997) produce credit cycles but not leverage cycles. In all those models, leverage is countercyclical despite that borrowing goes down after bad news. The reason for the discrepancy is that to generate leverage cycles, uncertainty is needed, and it is a particular type of uncertainty: one in which bad news is associated with an increase in future volatility. The literature on credit cycles has traditionally not been concerned with volatility. In our example above, leverage is the most important quantitative driver of the change in asset prices over the cycle. If the LTV were held to a constant, the cycle would be considerably dampened.

3.11. Leverage and Agent-Based Models

Recently, Thurner et al. (2012) reexamine the leverage cycle from an agent-based modeling perspective. In their model, fluctuations in volatility are entirely endogenous, rather than driven by shocks to asset dividends. It is assumed that the agents who leverage have a more stable opinion of the value of assets than do the cash buyers. When asset prices rise toward the value these leveraged buyers think is correct, their bets pay off, and they become relatively richer and come to control more of the market. Prices therefore become more stable, that is, volatility declines, so lenders permit borrowers to leverage more, driving volatility further down and their leverage further up. At that point, a little bit of bad news leads to margin calls and forced selling, which lead to rapid price declines and a spike in volatility. This causes lenders to toughen the margin requirements, creating more margin calls, more selling, and more volatility. In this agent-based model of the leverage cycle, it turns out that asset prices display clustered volatility and fat tails, even though all

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[18]In the static example of Section 2, we could have given both agents the same beliefs, $\gamma_U^b = 0.5$, provided that we gave them different endowments. If the beliefs are homogeneous and endowments for the O group are $e_0^O = 8.5$, $e_U^O = 4.85$, and $e_D^O = 42.5$ and for the P group are $e_0^P = 100$, $e_U^P = 125.1$, and $e_D^P = 83.26$, then we get the same equilibrium prices, collateral values, and liquidity wedge as we do in our example with different beliefs.

the shocks are essentially Gaussian. Details of the agent-based approach to leverage can be found in **Supplemental Appendix 4**.

4. MULTIPLE LEVERAGE CYCLES

Many kinds of collateral exist at the same time; hence, there can be many simultaneous leverage cycles. Collateral equilibrium theory not only explains how one leverage cycle might evolve over time, it also explains some commonly observed cross-sectional differences and linkages between cycles in different asset classes. When we extend the example in Section 3 to more than one asset, multiple coexisting leverage cycles can explain flight to collateral, contagion, and drastic swings in the volume of trade of high-quality assets. The technical details of this section, as well as complete numerical examples that show in detail how these cross-sectional properties arise, can be found in the **Supplemental Appendixes**.

4.1. Multiple Leverage Cycles and Flight to Collateral

When similar bad news hits two different asset classes, one asset class often preserves its value better than another. This empirical observation is traditionally given the name flight to quality because it is understood as a migration toward safer assets that have less volatile payoff values. Fostel & Geanakoplos (2008) emphasize a new channel that they call flight to collateral: After volatile bad news, collateral values widen more than payoff values, thus giving a different explanation for the diverging prices.

The example in **Supplemental Appendix 1** shows that flight to collateral arises in equilibrium when we extend the example in Section 3. We consider two perfectly correlated assets, Y and Z, where Y pays (1, 1, 1, 0.2) across states (UU, UD, DU, DD), as in Section 3, and Z pays (1, 1, 1, 0.1). At D, asset Y becomes safer than asset Z because 0.2 is greater than 0.1, but Y also becomes better collateral because it can back an equilibrium promise of 0.2, whereas Z can only back an equilibrium promise of 0.1.

In equilibrium, each asset experiences a leverage cycle. Prices for both assets go down after bad news by more than anybody thinks their expected values decline, just as in Section 3. However, something interesting happens when we look at the cross-sectional variation of all the variables. The gap between asset prices widens after bad news by more than the gap in expected payoffs. After bad news, the payoff value of Y goes down and that of Z goes down slightly more. However, their collateral values move in opposite directions: While the collateral value of Z goes down, falling in tandem with its payoff value and hence amplifying its leverage cycle, the collateral value of Y increases, mitigating the gravity of its leverage cycle. The spread in the prices of Y and Z grows by 0.034 at D, of which 0.001 results from the further spread between their payoff values and 0.033 results from the widening spread between their collateral values. What looks like a migration from Y to Z because Y is safer is actually a migration because Y is a better collateral.

Flight to collateral occurs when the liquidity wedge is high and the dispersion of LTVs is high. In the example, the liquidity wedge increases from 0 to D, and at D, Y can then be used to borrow 0.1 more dollars than one can borrow with Z, whereas at 0, Y can be used to borrow 0.043 more dollars. During a flight to collateral, when the liquidity wedge is high, investors would rather buy those assets that enable them to borrow more money (higher LTVs). Conversely, investors who need to raise cash get more by selling those assets on which they borrowed less money because the sales revenues net of loan repayments are higher.

Flight to collateral is related to what other papers call flight to liquidity. Flight to liquidity is discussed by Vayanos (2004) in a model in which an asset's liquidity is defined by its exogenously

given transaction cost. In Brunnermeier & Pedersen (2009), market liquidity is the gap between the fundamental value and the transaction price. They show how this market liquidity interacts with funding liquidity (given by trader's capital and margin requirements) generating flight to liquidity. In our model, an asset's liquidity is given by its capacity as collateral to raise cash. Hence, our flight to collateral arises from different leverage cycles in equilibrium and their interaction with the liquidity wedge cycle.

4.2. Multiple Leverage Cycles and Contagion

In this section, we show how multiple coexisting leverage cycles can explain contagion. When bad news hits one asset class, the resulting fall in its price can migrate to other assets, even if their payoffs are statistically independent from the original crashing assets.

In **Supplemental Appendix 2**, we extend the example in Section 3 to two independent assets, Y and Z. As in the original example, Y pays 1 for sure after good news U and 1 or 0.2 after bad news D. Conversely, Z pays off 1 or 0.2 after U and after D. In the extended example, bad news is about Y only. So we should expect the price of Y to go down after bad news owing to a deterioration of its expected payoff value. But we should not expect the price of asset Z to go down after bad news about Y.

In equilibrium, asset Y experiences a leverage cycle. But surprisingly, the price of Z also goes down after bad news about Y. Hence, the leverage cycle on Y migrates to asset class Z, inducing a pricing cycle on this market as well. In short, we see contagion in equilibrium.

Fostel & Geanakoplos (2008) show that contagion is generated by a change in the liquidity wedge. The Y leverage cycle lowers the liquidity wedge at U after good news and sharply increases the liquidity wedge at D after bad news, as seen in our previous examples. A leverage cycle in one asset class alone can move the liquidity wedge. But the liquidity wedge is a universal factor in valuing all assets, as seen in Section 2. An increase in the optimists' liquidity wedge after bad news reduces their valuation of all assets, including asset Z. There is also another factor that can be seen clearly in the example in **Supplemental Appendix 2**, which Fostel & Geanakoplos (2008) call the portfolio effect: Optimists see such a great opportunity at D that they end up holding more of asset Y after bad news than after good news, amplifying the movements of the liquidity wedge.

There is a vast literature on contagion. Despite the range of different approaches, there are mainly three different kinds of models. The first blends financial theories with macroeconomic techniques and seeks international transmission channels associated with macroeconomic variables. Examples of this approach include Corsetti et al. (1999) and Pavlova & Rigobon (2008). The second kind models contagion as information transmission. In this case, the fundamentals of assets are assumed to be correlated. When one asset declines in price because of noise trading, rational traders reduce the prices of all assets, as they are unable to distinguish declines due to fundamentals from declines due to noise trading. Examples of this approach include King & Wadhwani (1990), Calvo & Mendoza (2000), and Kodres & Pritsker (2002). Finally, there are theories that model contagion through wealth effects, as in Kyle & Xiong (2001). When some key financial actors suffer losses, they liquidate positions in several markets, and this sell-off generates price comovement. Our model shares with the last two approaches a focus exclusively on contagion as a financial market phenomenon. But our model further shows how leverage cycles can produce contagion in less extreme but more frequent market conditions: the anxious economy, in which there is no sell-off. The leverage cycle causes contagion even when trade patterns differ from those observed during acute crises.

4.3. Multiple Leverage Cycles, Adverse Selection, and the Volume of Trade

In this section we show that when we extend the model presented in Section 2 to encompass asymmetric information, multiple leverage cycles can generate violent swings in the volume of trade. **Supplemental Appendix 3** presents the extended model with endogenous leverage and adverse selection. Following Dubey & Geanakoplos (2002) and Fostel & Geanakoplos (2008), we can extend our collateral general equilibrium model to encompass asymmetric information. In the new model, investors are not endowed with assets. Assets are owned at first by a new class of agents called issuers. Importantly, issuers know the quality of their assets, but investors do not. The endogenous quantity signals are modeled in the same way endogenous leverage is modeled in Section 2. This strategy allows for signaling as well as adverse selection without destroying market anonymity.

Supplemental Appendix 3 also presents a numerical example extending the basic example of Section 3 to include two perfectly correlated assets of different quality and endogenous issuance under the presence of asymmetric information. In equilibrium, the price behavior described in Section 4.1 is still present here: There are two coexisting leverage cycles and flight to collateral. The new finding comes from the supply side. To signal that their assets are good (so investors will pay more for them and be able to borrow more using them as collateral), the issuers of the good-quality asset always sell less than they would if their types were common knowledge. However, after bad news at D, the drop in the volume of their sales is huge.

It is not surprising that with the bad news and the corresponding fall in prices, equilibrium issuance falls as well, because issuers are optimists and do not want to sell at such low prices. The interesting finding is that flight to collateral combined with informational asymmetries generates such a big drop in good issuance, even though the news is almost equally bad for both assets. The explanation is that the bigger price spread between types caused by the flight to collateral requires a smaller good type issuance for a separating equilibrium to exist. Unless the good issuance level becomes onerously low, bad types would be more tempted by the bigger price spread to mimic good types and sell at the high price. The good types are able to separate themselves by choosing low-enough quantities, as it is more costly for the bad types to rely on the payoff of their own asset for final consumption than it is for the good types.

There is a growing literature that tries to model asymmetric information within a general equilibrium, including Gale (1992), Bisin & Gottardi (2006), and Rustichini & Siconolfi (2008). Our model combines asymmetric information in a general equilibrium model with a model of endogenous credit constraints and leverage.

LITERATURE CITED

Acharya V, Gale D, Yorulmazer T. 2011. Rollover risk and market freezes. *J. Finance* 66:1177–209

Acharya V, Viswanathan S. 2011. Leverage, moral hazard, and liquidity. *J. Finance* 66:99–138

Adrian T, Boyarchenko N. 2012. *Intermediary leverage cycles and financial stability*. Staff Rep. 567, Fed. Reserve Bank New York

Adrian T, Shin H. 2010. Liquidity and leverage. *J. Financ. Intermed*. 19:418–37

Araujo A, Kubler F, Schommer S. 2012. Regulating collateral-requirements when markets are incomplete. *J. Econ. Theory* 147:450–76

Bernanke B, Gertler M. 1989. Agency costs, net worth, and business fluctuations. *Am. Econ. Rev*. 79:14–31

Bisin A, Gottardi P. 2006. Efficient competitive equilibria with adverse selection. *J. Polit. Econ*. 114:485–516

Brunnermeier M. 2009. Deciphering the liquidity and credit crunch 2007–2008. *J. Econ. Perspect*. 23(1):77–100

Brunnermeier M, Pedersen L. 2009. Market liquidity and funding liquidity. *Rev. Financ. Stud*. 22:2201–38

Brunnermeier M, Sannikov Y. 2014. A macroeconomic model with a financial sector. *Am. Econ. Rev*. In press

Calvo G, Mendoza E. 2000. Contagion, globalization, and the volatility of capital flows. In *Capital Flows and the Emerging Economies: Theory, Evidence, and Controversies*, ed. S Edwards, pp. 15–41. Chicago: Univ. Chicago Press

Corsetti G, Pesenti P, Roubini N. 1999. Paper tigers? A model of the Asian crisis. *Eur. Econ. Rev*. 43:1211–36

Dubey P, Geanakoplos J. 2002. Competitive pooling: Rothschild-Stiglitz reconsidered. *Q. J. Econ*. 117:1529–70

Fostel A, Geanakoplos J. 2008. Leverage cycles and the anxious economy. *Am. Econ. Rev*. 98:1211–44

Fostel A, Geanakoplos J. 2012a. Tranching, CDS and asset prices: how financial innovation can cause bubbles and crashes. *Am. Econ. J. Macroecon*. 4(1):190–225

Fostel A, Geanakoplos J. 2012b. Why does bad news increase volatility and decrease leverage? *J. Econ. Theory* 147:501–25

Fostel A, Geanakoplos J. 2013. *Leverage and default in binomial economies: a complete characterization*. Work. Pap. 1877R, Cowles Found., Yale Univ., New Haven, CT

Gale D. 1992. A Walrasian theory of markets with adverse selection. *Rev. Econ. Stud*. 59:229–55

Garleanu N, Pedersen L. 2011. Margin-based asset pricing and the law of one price. *Rev. Financ. Stud*. 24:1980–2022

Geanakoplos J. 1997. Promises, promises. In *The Economy as an Evolving Complex System II*, ed. WB Arthur, S Durlauf, D Lane, pp. 285–320. Reading, MA: Addison-Wesley

Geanakoplos J. 2003. Liquidity, default, and crashes: endogenous contracts in general equilibrium. In *Advances in Economics and Econometrics*, Vol. 2: *Theory and Applications, Eighth World Conference*, ed. M Dewatripont, LP Hansen, SJ Turnovsky, pp. 170–205. Cambridge, UK: Cambridge Univ. Press

Geanakoplos J. 2010. The leverage cycle. In *NBER Macroeconomics Annual 2009*, ed. D Acemoglu, K Rogoff, M Woodford, pp. 1–65. Chicago: Univ. Chicago Press

Geanakoplos J, Kubler F. 2013. *Why is too much leverage bad for the economy?* Work. Pap., Cowles Found., Yale Univ., New Haven, CT

Geanakoplos J, Polemarchakis H. 1986. Existence, regularity, and constrained suboptimality of competitive allocations when the asset market is incomplete. In *Essays in Honor of Kenneth Arrow*, Vol. 3, ed. W Heller, R Starr, D Starrett, pp. 65–95. Cambridge, UK: Cambridge Univ. Press

Geanakoplos J, Zame WR. 1997. *Collateralized security markets*. Work. Pap., Cowles Found., Yale Univ., New Haven, CT

Geanakoplos J, Zame WR. 2014. Collateral equilibrium, I: a basic framework. *Econ. Theory*. In press

Gorton G. 2009. The subprime panic. *Eur. Financ. Manag*. 15:10–46

Gromb D, Vayanos D. 2002. Equilibrium and welfare in markets with financially constrained arbitrageurs. *J. Financ. Econ*. 66:361–407

Harrison JM, Kreps D. 1978. Speculative investor behavior in a stock market with heterogeneous expectations. *Q. J. Econ*. 92:323–36

Hindy A. 1994. Viable prices in financial markets with solvency constraints. *J. Math. Econ*. 24(2):105–35

Holmstrom B, Tirole J. 1997. Financial intermediation, loanable funds, and the real sector. *Q. J. Econ.* 112:663–91

King M, Wadhwani S. 1990. Transmission of volatility between stock markets. *Rev. Financ. Stud.* 3:5–33

Kiyotaki N, Moore J. 1997. Credit cycles. *J. Polit. Econ.* 105:211–48

Kodres L, Pritsker M. 2002. A rational expectations model of financial contagion. *J. Finance* 57:768–99

Kyle A, Xiong W. 2001. Contagion as a wealth effect. *J. Finance* 56:1401–40

Lagos R. 2010. Asset prices and liquidity in an exchange economy. *J. Monet. Econ.* 57:913–30

Mendoza E. 2010. Sudden Stops, financial crises, and leverage. *Am. Econ. Rev.* 100:1941–66

Minsky H. 1986. *Stabilizing an Unstable Economy*. New Haven, CT: Yale Univ. Press

Pavlova A, Rigobon R. 2008. The role of portfolio constraints in the international propagation of shocks. *Rev. Econ. Stud.* 75:1215–56

Reinhart C, Rogoff K. 2009. *This Time Is Different*. Princeton, NJ: Princeton Univ. Press

Rustichini A, Siconolfi P. 2008. General equilibrium in economies with adverse selection. *Econ. Theory* 37:417–37

Simsek A. 2013. Belief disagreements and collateral constraints. *Econometrica* 81:1–53

Smets F, Wouters R. 2007. Shocks and frictions in US business cycles: a Bayesian DSGE approach. *Am. Econ. Rev.* 97:586–606

Thurner S, Farmer D, Geanakoplos J. 2012. Leverage causes fat tails and clustered volatility. *Quant. Finance* 12:695–707

Vayanos D. 2004. *Flight to quality, flight to liquidity, and the pricing of risk*. NBER Work. Pap. 10327

Vayanos D, Wang J. 2012. Liquidity and asset prices under asymmetric information and imperfect competition. *Rev. Financ. Stud.* 25:1339–65