FINANCIAL INNOVATIONS, COLLATERAL, AND INVESTMENT

by

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Financial innovations that change how promises are collateralized affect prices and investment, even in the absence of any change in fundamentals. In C-models, the ability to leverage an asset always generates overinvestment compared to Arrow-Debreu. Credit Default Swaps always leads to underinvestment with respect to Arrow-Debreu, and in some cases even robustly destroy competitive equilibrium. The need for collateral would seem to cause underinvestment. Our analysis illustrates a countervailing force: goods that serve as collateral yield additional services and can therefore be over-valued and over-produced. In models without cash flow problems there is never marginal underinvestment on collateral. (JEL D52, D86, D92, E44, G01, G12, R31)

After the recent subprime crisis and the sovereign debt crisis in the euro zone, many observers have placed financial innovations such as leverage and credit default swaps (CDS) at the root of the problem.[1] Figure 1 shows how the financial crisis in the US was preceded by years in which leverage, prices, and investment increased dramatically in the US housing market and all collapsed together after the crisis. Figure 2 shows that CDS was a financial innovation introduced much later than leverage. Figure 3 shows how the peak in CDS volume coincides with the crisis and the crash in prices and investment in the US housing market.[2]

In this paper, we build a special class of models, that we call C and C^*-models, in which markets are incomplete and agents need to post collateral to back promises. First, we show that a financial innovation that enables agents to leverage the risky asset (i.e., to use it as collateral to back loans), will raise risky asset prices and investment. Second, and more surprisingly, when unlimited leverage is available, the market will endogenously choose leverage that leads to risky asset prices and investment above the Arrow-Debreu first-best level. Paradoxically, the requirement to post collateral in order to get a loan can lead to overinvestment compared to a

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[1] See for example Brunneimeier (2009); Geanakoplos (2010); Gorton (2009); and Stulz (2009). Geanakoplos (2003) and Fostel and Geanakoplos (2008) wrote before the crisis.

[2] The available numbers on CDS volumes are not specific to mortgages, since most CDS were over the counter, but the fact that subprime CDS were not standardized until late 2005 suggests that the growth of mortgage CDS in 2006 is likely even sharper than Figure 3 suggests.

Panel A. Leverage and prices in US housing market
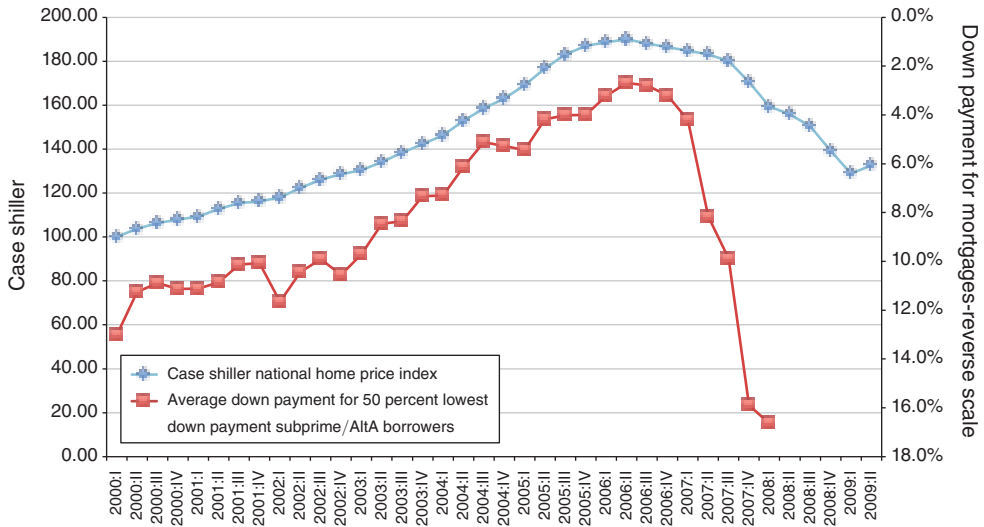
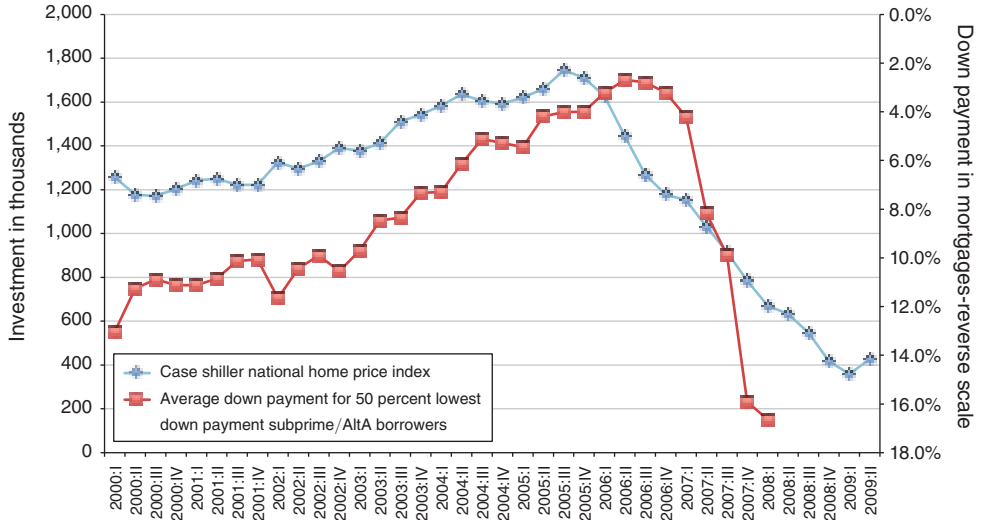

Panel B. Leverage and investment in US housing market

FIGURE 1

Notes: Observe that the down payment axis has been reversed because lower down payment requirements are correlated with higher home prices. For every AltA or subprime first loan originated from 2000:I to 2008:I, down payment percentage was calculated as appraised value (or sale price if available) minus total mortgage debt, divided by appraised value. For each quarter, the down payment percentages were ranked from highest to lowest, and the average of the bottom half of the list is shown in the diagram. This number is an indicator of down payment required: clearly many homeowners put down more than they had to, and that is why the top half is dropped from the average. A 13 percent down payment in 2000:I corresponds to leverage of about 7.7, and a 2.7 percent down payment in 2006:II corresponds to leverage of about 37. Note Subprime/AltA Issuance stopped in 2008:I.

Source: Panel A: Geanakoplos (2010). Panel B: Investment: Construction new privately owned housing units completed. Department of Commerce.
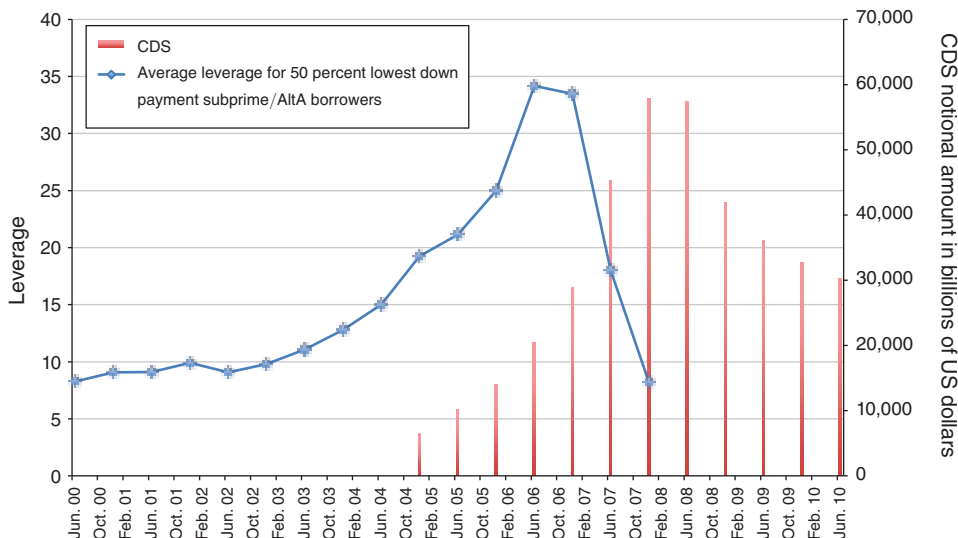
FIGURE 2. LEVERAGE AND CREDIT DEFAULT SWAPS

Notes: Observe that the down payment axis has been reversed because lower down payment requirements are correlated with higher home prices. For every AltA or subprime first loan originated from 2000:I to 2008:I, down payment percentage was calculated as appraised value (or sale price if available) minus total mortgage debt, divided by appraised value. For each quarter, the down payment percentages were ranked from highest to lowest, and the average of the bottom half of the list is shown in the diagram. This number is an indicator of down payment required: clearly many homeowners put down more than they had to, and that is why the top half is dropped from the average. A 13 percent down payment in 2000:I corresponds to leverage of about 7.7; and a 2.7 percent down payment in 2006:II corresponds to leverage of about 37. Note subprime/AltA Issuance stopped in 2008:I.

Source: CDS: Bank for International Studies (BIS) over-the-counter (OTC) Derivatives Market Statistics

first-best world in which borrowing is not limited by collateral. Third, when in the leverage economy a new financial innovation enables agents to use the riskless asset as collateral to back a CDS on the risky asset, risky asset prices and investment will fall not only below the leverage level, but all the way below the first-best level.

Needless to say, outside C^*-models it may not be the case that collateral restrictions lead to higher prices and investment than in Arrow-Debreu. Therefore, we seek a principle for general economies that describes the effect of collateral on prices and investment. We prove that the need to post collateral always generates marginal overinvestment: a collateral constrained agent who is given the opportunity to borrow a dollar without posting collateral will always spend the money on consumption, and never on investment projects that could be collateralized.

The central element of our analysis is *repayment enforceability problems*: we suppose that agents cannot be coerced into honoring their promises except by seizing collateral agreed upon by contract in advance. Agents need to post collateral in order to issue promises. To sharpen our conclusions, we explicitly assume away *collateral cash flow problems*. We suppose that all of the future value of collateral can be pledged: every agent knows exactly how the collateral cash flow depends on the future state of nature, and the cash flow is independent of how the investment was financed. This eliminates any issues associated with hidden effort or unobservability.

Panel A. CDS and prices in the US housing market
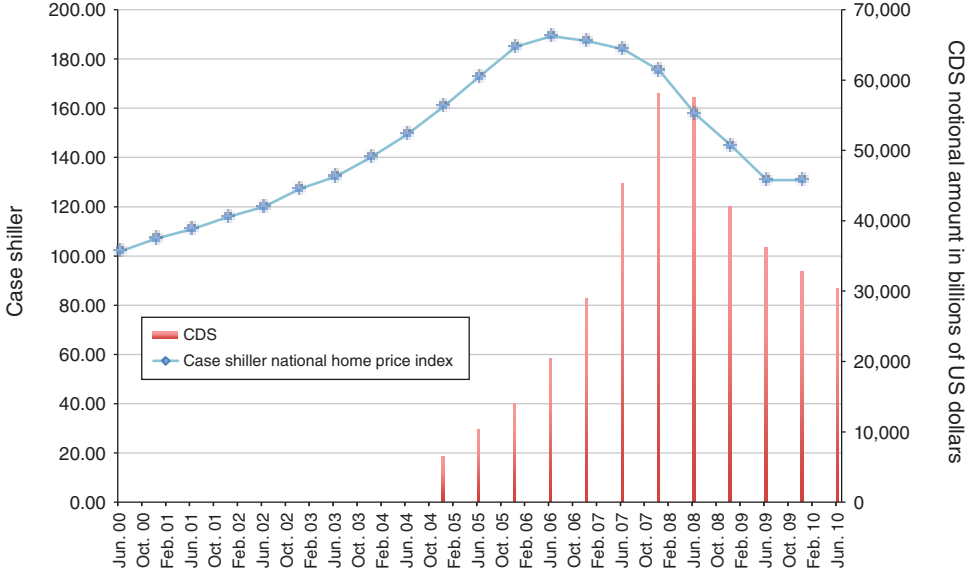
Panel B. CDS and investment in the US housing market
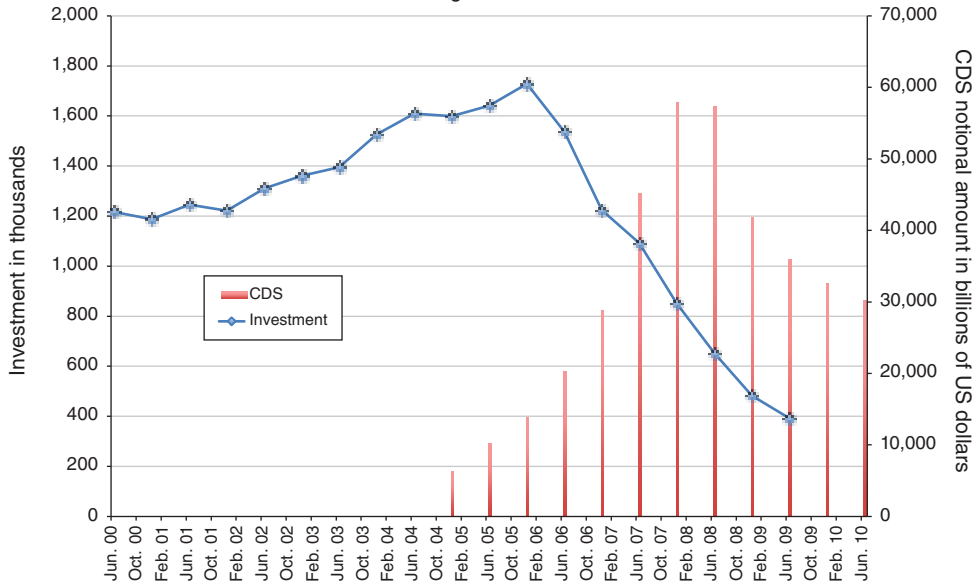
FIGURE 3

Source: CDS: BIS OTC Derivatives Market Statistics. Investment: Construction new privately owned housing units completed. Department of Commerce

We define a financial contract as an ordered pair consisting of a promise and the collateral backing it. The collection of available financial contracts is denoted by *J*. We define financial innovation as the use of new kinds of collateral, or new kinds of promises that can be backed by collateral, that is, by a change in *J*. In the incomplete markets literature, financial innovations were modeled by securities with new kinds

of payoffs. Financial innovations of that kind do have an effect on asset prices and real allocations, but the direction of the consequences is typically ambiguous and therefore has not been much explored. When we model financial innovation taking into account collateral, we can prove unambiguous results.

We examine four *J*-economies. In the first, there are no financial contracts, which we call financial autarky or the *N*-economy. In the second, we suppose that financial innovation has enabled agents to issue noncontingent promises (loans) using assets as collateral, but not to sell short or to issue contingent promises. We call this the leverage or *L*-economy. Into the previous leverage economy we introduce CDS on risky assets collateralized by the riskless asset, since cash is generally used as collateral for sellers of CDS. We call this the *CDS*-economy. Finally, we suppose that every contingent promise can be pledged by future endowments, which gives the first-best Arrow-Debreu economy.

We consider three different collateral models. The first C^*-model involves just two states of nature (U and D), a risky asset, and a riskless asset that can be used to produce the risky asset, in which all future consumption is derived from asset payoffs. A special case of the latter called the *C*-model, introduced by Geanakoplos (2003), has a continuum of risk neutral agents with heterogenous beliefs. The *C*-model is complex enough to allow for the possibility that financial innovation can have a big effect on prices and investment. But it is simple enough to be tractable (via an Edgeworth Box diagram) and to generate unambiguous (as well as intuitive) results. For this reason, in most of the paper we conduct our analysis in *C*-models and point out when our results can be extended to C^*-models. Finally, we consider a completely general model of collateral equilibrium that can encompass arbitrary preferences, endowments, assets, consumption goods, and heterogeneous technologies in order to study our general property of marginal over investment.

We prove that in *C* and C^*-models, investment is always higher in the *L*-economy than in the *N*-economy. Thus, leverage raises the risky asset price and investment. More surprisingly, we show that this ability to leverage the risky asset generates overvaluation and overinvestment compared to the Arrow-Debreu level. Here is one intuition for this result. Agents who are borrowing constrained will prefer to hold assets that provide collateral so that they can simultaneously borrow. This collateral value appears even when there is no uncertainty. When there is uncertainty, the ability to leverage an asset provides an opportunity to place a bigger bet on the asset rising in value, generating a speculative demand for the asset. This ability to bet increases the collateral value of the risky asset even further. Agents thus have more incentive to produce goods that are better collateral. In an Arrow-Debreu economy, agents can borrow without holding any particular asset, and they can bet by buying or selling Arrow securities, again without needing to buy any asset. Finally, we show that when production displays constant returns to scale, the Arrow-Debreu equilibrium Pareto dominates the leverage equilibrium, which, in turn, Pareto dominates the financial autarky equilibrium.

We also prove that in the *C*-model, investment dramatically falls in the *CDS*-economy, not only below the *L*-economy level, but beneath the Arrow-Debreu level. CDS decreases investment in the risky asset because the seller of CDS is

effectively making the same kind of investment as the buyer of the leveraged risky asset: she obtains a portfolio of the riskless asset as collateral and the CDS obligation, which on net pays off precisely when the asset does very well, just like the leveraged purchase. The creation of CDS thus lures away the speculative demand for the risky asset, creating a collateral value for the riskless asset.

Finally, we show that the creation of CDS may in fact destroy equilibrium by choking off all production. CDS is a derivative, whose payoff depends on some underlying asset. The quantity of CDS that can be traded is not limited by the market size of the underlying risky asset.[3] If the production of the underlying asset diminishes, the CDS trading may continue at the same high levels. But when the production of the underlying risky asset falls to zero, CDS trading must come to an end by definition. This discontinuity can cause robust nonexistence. However, in regions in which the CDS equilibrium exists, we cannot establish unambiguous welfare comparisons between the *CDS*-economy and the Arrow-Debreu economy.

Another way of understanding these results is as follows. When agents post collateral to back promises they are effectively tranching the collateral cash flows. By dividing up the collateral payoffs into two different kinds of assets, attractive to two different clienteles, demand is increased. This leads to a higher collateral value. In the case of leverage, the risky asset cash flows are tranched into an Arrow U (attractive to optimistic buyers) and a riskless bond (attractive to the general public). Similarly, CDS can be thought of as a sophisticated tranching of the riskless asset. A CDS tranches the riskless asset cash flows into an Arrow U and an Arrow D. This tends to raise the collateral value of the riskless asset relative to the collateral value of the risky asset, thereby reducing the production of the risky asset. As before, investment migrates to good collateral.

When restricting ourselves to the special class of C and C^*-models, we can generate sharp results. However, our results comparing collateral equilibrium with Arrow-Debreu equilibrium are bound not to be general.[4] For this reason, in the second part of our analysis we identify a completely general phenomenon, which applies to any commodity that can serve as collateral for any kind of promise, provided there are no cash flow problems. We replace the Arrow-Debreu benchmark with a local concept of efficiency. If agents are really underinvesting because they are borrowing constrained, then if presented with a little bit of extra money to make a purely cash purchase, they should invest. Yet we prove in a general model with arbitrary preferences and states of nature that none of them would choose to produce more of any good that can be used as collateral, even if they were also given access to the best technology available in the economy. Thus without cash flow problems, repayment enforceability problems can lead to

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[3] Currently the outstanding notional value of CDS in the United States is far in excess of $50 trillion, more than three times the value of their underlying assets.

[4] For instance, with risk neutral agents, if we change endowments in the future, collateral equilibrium would not change, since future endowments cannot be used as collateral, but the Arrow-Debreu equilibrium would. So by playing with future endowments it would be easy to create an example in which the L-equilibrium has less investment than the Arrow-Debreu equilibrium. This could not occur in C-models because we supposed that all future consumption is derived from dividends of assets existing from the beginning.

marginal overinvestment, but never marginal underinvestment. In C and C^*-models, the marginal overinvestment is big enough to exceed the Arrow-Debreu level.

Our overinvestment result may seem surprising to the reader, since it stands in contrast with the traditional macroeconomic/corporate finance literature with financial frictions, such as in Bernanke and Gertler (1989) and Kiyotaki and Moore (1997). In these papers financial frictions generate underinvestment with respect to Arrow-Debreu. Their result may appear intuitive since one would expect that the need for collateral would prevent some investors from borrowing the money to invest, thus reducing production. In our model borrowers may also find themselves constrained: they cannot borrow more at the same interest rate on the same collateral. Yet we show that in C and in C^*-models there is never underinvestment with respect to Arrow-Debreu. There are two reasons for the discrepancy. First, the traditional literature did not recognize (or at least did not sufficiently emphasize) the collateral value of assets that can back loans. Precisely because agents are constrained in what they can borrow, they will overvalue commodities that can serve as collateral (compared to perishable consumption goods or other commodities that cannot), which might lead to overproduction of these collateral goods. The second reason for the discrepancy is that in the macro/corporate finance models, it is assumed that borrowers cannot pledge the whole future value of the assets they produce. In other words, these papers are explicitly considering what we here call *collateral cash flow problems.*[5] When we disentangle the *cash flow problems* from the *repayment enforceability problems* we get the opposite result: there can be overinvestment even when agents are constrained in their borrowing. With our modeling strategy we expose a countervailing force in the incentives to produce: when only some assets can be used as collateral, they become relatively more valuable, and are therefore produced more.

Our results show that the traditional macroeconomic/financial frictions literature overlooked the possibility that the need for collateral may be useful in explaining booms in collateralizable investments as well as in explaining busts. On the other hand, our model displays the same potential for underinvestment in noncollateralizable projects as the traditional literature. One way to move from overinvestment to underinvestment is to suppose that a good could be fully collateralized at some point, and then becomes prohibited from being used as collateral at another. Many subprime mortgages went from being prominent collateral on Repo in 2006 to being not accepted as collateral in 2009. Another transition emerges when leverage endogenously falls on good collateral, preventing agents from borrowing as much on good collateral to invest in uncollateralized projects. Between 2003 and 2007, American households increased their borrowing on homes they already owned by $1 trillion per year, spending much of it on nonhousing related projects. After 2007 cash extraction from housing almost entirely disappeared.

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[5] In Kiyotaki and Moore (1997), the lender cannot confiscate the fruit growing on the land but just the land. Other examples of cash flow problems are to be found in corporate finance asymmetric information models such as Holmstrom and Tirole (1997), Adrian and Shin (2010), and Acharya and Viswanathan (2011). The idea in this literature is that collateral payoffs deteriorate if too much money is borrowed, because then the owner has less incentive to work hard to obtain good cash flows.

In this paper we follow the model of collateral equilibrium developed in Geanakoplos (1997, 2003, 2010); Fostel and Geanakoplos (2008, 2012a and 2012b, 2014, forthcoming); and Geanakoplos-Zame (2014). Geanakoplos (2003) showed that leverage can raise asset prices. Geanakoplos (2010) and Che and Sethi (2010) showed that in the kind of models studied by Geanakoplos (2003), CDS can lower risky asset prices. Fostel-Geanakoplos (2012b) showed more generally how different kinds of financial innovations can have big effects on asset prices. In this paper, we move a step forward and show that financial innovation affects investment as well.

Our model is related to a literature on financial innovation pioneered by Allen and Gale (1994), though in our paper financial innovation is taken as given, and concerns collateral. There are other macroeconomic models with financial frictions such as Kilenthong and Townsend (2011) that produce overinvestment in equilibrium. The underlying mechanism in these papers is very different from the one presented in our paper. In those papers the overinvestment is due to an externality through changing relative prices in the future states. Our results do not rely on relative price changes in the future, and to make the point clear we restrict our C and C^*-models to a single consumption good in every future state.

Our paper is also related to Polemarchakis and Ku (1990). They provide a robust example of nonexistence in a general equilibrium model of exchange with incomplete markets, due to the presence of derivatives. Existence had been proved to be generic in the canonical general equilibrium model of exchange with incomplete markets and no derivatives by Duffie and Shaffer (1986). Geanakoplos and Zame (2014) proved that equilibrium always exists in pure exchange economies even with derivatives if there is a finite number of potential contracts, with each requiring collateral. Thus, the need for collateral to enforce deliveries on promises eliminates the nonexistence problem in pure exchange economies with derivatives such as in Polemarchakis-Ku. Our paper gives a robust example of nonexistence in a general equilibrium model with incomplete markets and collateral, production, and derivatives. Thus, the nonexistence problem emerges again with derivatives and production, despite the collateral.

The paper is organized as follows. Section I presents the collateral general equilibrium model and the special class of C and C^*-models. Section II presents numerical examples and our propositions for the C and C^*-models. Section III discusses the nonexistence result. Section IV introduces the notion of marginal efficiency and presents the marginal overinvestment results in the general model of Section II. Section V characterizes equilibrium in C-models for different financial innovations. It also presents a novel use of an Edgeworth Box diagram for trade with a continuum of agents (rather than just two agents) with heterogeneous but linear preferences, that we use to prove all our propositions from Section II for the C-model. It presents yet another diagram to prove the extension of some of the propositions to C^*-models.

Appendices A–C present supplemental material.

I. Collateral General Equilibrium Model

In this section, we present the collateral general equilibrium model and a special class of collateral models that we call the C-model and C^*-model, which will be extensively used in the paper.[6]

A. *Time and Commodities*

Consider a two-period general equilibrium model, with time $t = 0, 1$. Uncertainty is represented by different states of nature $s \in S$ including a root $s = 0$. We denote the time of s by $t(s)$, so $t(0) = 0$ and $t(s) = 1, \forall s \in S_T$, the set of terminal nodes of S. Suppose there are L_s commodities in $s \in S$. Let the set of all commodities be denoted by $L = \times_{s \in S} L_s$, and the set of all terminal commodities be denoted by $L_T = \times_{s \in S_T} L_s$. Let $\mathbf{p}_s \in R_+^{L_s}$ denote the vector of commodity prices in each state $s \in S$.

B. *Agents*

Each investor $h \in H$ is characterized by Bernoulli utilities, u_s^h, $s \in S$, a discount factor, β_h, and subjective probabilities, γ_s^h, $s \in S_T$. The utility function for commodities in $s \in S$ is $u_s^h : R_+^{L_s} \to R$, and we assume that these state utilities are differentiable, concave, and weakly monotonic (more of every good in any state strictly improves utility). The expected utility to agent h is:

$$(1) \qquad U^h = u_0^h(\mathbf{x}_0) + \beta_h \sum_{s \in S_T} \gamma_s^h u_s^h(\mathbf{x}_s).$$

Investor h's endowment of the commodities is denoted by $\mathbf{e}_s^h \in R_+^{L_s}$ in each state $s \in S$.

C. *Production*

For each $s \in S$ and $h \in H$, let $Z_s^h \subset \mathbb{R}^{L_s}$ denote the set of feasible intra-period production for agent h. Commodities can enter as inputs and outputs of the intra-period production process; inputs appear as negative components $z_l < 0$ of $\mathbf{z} \in Z_s^h$, and outputs as positive components $z_l > 0$ of $\mathbf{z} \in Z_s^h$. We assume that Z_s^h is convex, compact and that $\mathbf{0} \in Z_s^h$.

We also allow for inter-period production. For each $h \in H$, let $F^h : \mathbb{R}_+^{L_0} \to \mathbb{R}_+^{L_T}$ be a linear inter-period production function connecting a vector of commodities $\mathbf{x}_0$ at state $s = 0$ with the vector of commodities $\mathbf{F}_s^h(\mathbf{x}_0) \in \mathbb{R}_+^{L_s}$ it becomes in each state $s \in S_T$.

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[6] The C-model was introduced in Geanakoplos (2003).

Production enables our model to include many different kinds of commodities. Commodities could either be perishable consumption goods (like food), or durable consumption goods (like houses), or they could represent assets (like Lucas trees) that pay dividends. The holder of a durable consumption good can enjoy current utility as well as the prospect of the future realization of the goods (either by consuming them or selling them). The buyer of a durable asset can expect the income from future dividends.

D. *Financial Contracts and Collateral*

The heart of our analysis involves financial contracts and collateral. We explicitly incorporate *repayment enforceability problems*, but exclude *cash flow problems*. Agents cannot be coerced into honoring their promises except by seizing collateral agreed upon by contract in advance. Agents need to post collateral in the form of durable assets in order to issue promises. But there is no doubt what the collateral will pay, conditional on the future state of nature.

A *financial contract j* promises $\mathbf{j}_s \in R_+^{L_s}$ commodities in each final state $s \in S_T$ backed by collateral $\mathbf{c}_j \in R_+^{L0}$. This allows for noncontingent promises of different sizes, as well as contingent promises. The price of contract j is π_j. Let θ_j^h be the number of contracts j traded by h at time 0. A positive θ_j^h indicates agent h is buying contracts j or lending $\theta_j^h \pi_j$. A negative θ_j^h indicates agent h is selling contracts j or borrowing $\left|\theta_j^h\right| \pi_j$.

We wish to exclude collateral cash flow problems, stemming, for example, from adverse selection or moral hazard, beyond repayment enforceability problems. Accordingly, we eliminate adverse selection by restricting the sale of each contract j to a set $H(j) \subset H$ of traders with the same durability functions, $\mathbf{F}^h(\mathbf{c}_j) = \mathbf{F}^{h'}(\mathbf{c}_j)$ if $h, h' \in H(j)$. Since we assumed that the most borrowers can lose is their collateral if they do not honor their promise, the actual delivery of contract j in states $s \in S_T$ is

$$(2) \qquad \delta_s(j) = \min\left\{\mathbf{p}_s \cdot \mathbf{j}_s, \mathbf{p}_s \cdot \mathbf{F}_s^{H(j)}(\mathbf{c}_j)\right\}.$$

Notice that there are no cash flow problems: the value of the collateral in each future state does not depend on the size of the promise, or on what other choices the seller $h \in H(j)$ makes, or on who owns the asset at the very end. This eliminates any issues associated with hidden effort or unobservability.

A final assumption that we will consider in Section VI to eliminate cash flow problems is to suppose that promises are not artificially limited. We suppose that if $\mathbf{c}_j$ is the collateral for some contract j, then there is a "large" contract j' with $\mathbf{c}_{j'} = \mathbf{c}_j$ and $H(j') = H(j)$ and $\mathbf{j}_s' \geq \mathbf{F}_s^{H(j)}(\mathbf{c}_j)$ for all $s \in S_T$.

E. *Budget Set*

Given commodity and financial contract prices $\left(\mathbf{p}, \left(\pi_j\right)_{j \in J}\right)$, each agent $h \in H$ chooses production, $\mathbf{z}_s$, and commodities, $\mathbf{x}_s$, for each $s \in S$, and contract trades, θ_j, at time 0, to maximize utility (1) subject to the budget set defined by

$$
B^h(\mathbf{p}, \pi) = \Big\{ (\mathbf{z}, \mathbf{x}, \boldsymbol{\theta}) \in R^L \times R_+^L \times \left(R^J\right) :
$$

$$
\mathbf{p}_0 \cdot (\mathbf{x}_0 - \mathbf{e}_0^h - \mathbf{z}_0) + \sum_{j \in J} \theta_j \pi_j \leq 0
$$

$$
\mathbf{p}_s \cdot (\mathbf{x}_s - \mathbf{e}_s^h - \mathbf{z}_s) \leq \mathbf{p}_s \cdot \mathbf{F}_s^h(\mathbf{x}_0) + \sum_{j \in J} \theta_j \min\left\{ \mathbf{p}_s \cdot \mathbf{j}_s, \mathbf{p}_s \cdot \mathbf{F}_s^{H(j)}(\mathbf{c}_j) \right\}, \quad \forall s \in S_T
$$

$$
\mathbf{z}_s \in Z_s^h, \forall s \in S
$$

$$
\theta_j < 0 \text{ only if } h \in H(j)
$$

$$
\sum_{j \in J} \max(0, -\theta_j) \mathbf{c}_j \leq \mathbf{x}_0 \Big\}.
$$

The first inequality requires that money spent on commodities beyond the revenue from endowments and production in state 0 be financed out of the sale of contracts. The second inequality requires that money spent on commodities beyond the revenue from endowments and production in any state $s \in S_T$ be financed out of net revenue from dividends from contracts bought or sold in state 0. The third constraint requires that production is feasible, the fourth constraint requires that only agents $h \in H(j)$ can sell contract j, and the last constraint requires that agent h actually holds at least as much of each good as she is required to post as collateral.

F. *Collateral Equilibrium*

A *Collateral Equilibrium* is a set of commodity prices, contract prices, production and commodity holdings, and contract trades $\left((\mathbf{p}, \pi), (\mathbf{z}^h, \mathbf{x}^h, \boldsymbol{\theta}^h)_{h \in H}\right) \in R_+^L \times R_+^J \times \left(R^L \times R_+^L \times R^J\right)^H$, such that

(i) $\sum_{h \in H} \left(\mathbf{x}_0^h - \mathbf{e}_0^h - \mathbf{z}_0^h\right) = \mathbf{0}.$

(ii) $\sum_{h \in H} \left(\mathbf{x}_s^h - \mathbf{e}_s^h - \mathbf{z}_s^h - \mathbf{F}_s^h\left(\mathbf{x}_0^h\right)\right) = \mathbf{0}, \quad \forall s \in S_T.$

(iii) $\sum_{h \in H} \theta_j^h = 0, \quad \forall j \in J.$

(iv) $\left(\mathbf{z}^h, \mathbf{x}^h, \boldsymbol{\theta}^h\right) \in B^h(\mathbf{p}, \pi), \quad \forall h$

(v) $(\mathbf{z}, \mathbf{x}, \boldsymbol{\theta}) \in B^h(\mathbf{p}, \pi) \Rightarrow U^h(\mathbf{x}) \leq U^h\left(\mathbf{x}^h\right), \quad \forall h.$

Markets for consumption in state 0 and in states $s \in S_T$ clear, as do contract markets. Furthermore, agents optimize their utility in their budget set. Geanakoplos and Zame (2014) show that collateral equilibrium always exists in a very similar economy but without production. Their argument can be extended to encompass our collateral model with production.

G. *Financing Investment and the Firm*

Let us pause for a moment to consider how investment is financed, and how we could think of a firm in our model.

Agent h finances inter-period production $\mathbf{F}^h(\mathbf{y})$ by purchasing the inputs $\mathbf{y} \in R_+^{L0}$ partly out of money borrowed, by issuing a contract j using $\mathbf{y} = \mathbf{c}_j$ as collateral, and the rest out of endowment income $\mathbf{p}_0 \cdot \mathbf{e}_0^h$. Agent h finances intra-period production $\mathbf{z}_s^h$ by selling the output $\max(\mathbf{0}, \mathbf{z}_s^h)$ in advance to the buyers, and then using the proceeds to buy the inputs $\max(\mathbf{0}, -\mathbf{z}_s^h)$ needed to produce the output, just like a home builder who lines up the owner before she begins construction.

There are three possible interpretations of a firm in our model. In the first, the firm is defined by F^h; in this case the firm is directly affected by limitations to financing. In the second interpretation, the firm is defined by Z_s^h. In this case, the firm does not directly face any financing restrictions, but the consumer/buyer of its output does, indirectly affecting the firm's investment decision. In this interpretation, we emphasize consumer durables and the collateral constraint affecting the consumer.

In the third interpretation, production takes two periods, and the firm is characterized by $F^h \circ Z_0^h$. In this interpretation, the intra-period output could be interpreted as intangible but irrevocable plans to produce. Once these plans are in place there is no doubt about the future output $F^h(\mathbf{z}_0^+)$. Now the firm itself is the collateral for any borrowing.

H. *C^*-Economies and C-Economies*

The C^*-model is defined as follows. We consider a binary tree, so that $S = \{0, U, D\}$. In states U and D there is a single commodity, called the consumption good, and in state 0 there are two commodities, called assets X and Y. We take the price of the consumption good in each state U, D to be 1, and the price of X to be 1 at time 0. We denote the price of asset Y at time 0 by p.

The riskless asset X yields dividends $d_U^X = d_D^X = 1$ unit of the consumption good in each state, and the risky asset Y pays d_U^Y units of the consumption good in state U and $0 < d_D^Y < d_U^Y$ units of the consumption good in state D. Formally, inter-period production is defined as $F_U^h(X, Y) = F_U(X, Y) = d_U^X X + d_U^Y Y = X + d_U^Y Y$ and $F_D^h(X, Y) = F_D(X, Y) = d_D^X X + d_D^Y Y = X + d_D^Y Y$. Since inter-period production is the same for each agent, we take $H(j) = H$ for all contracts $j \in J$.

The intra period technology at 0, $Z_0^h = Z_0 \subset \mathbb{R}^2$, is also the same for all agents, and allows each of them to invest the riskless asset X and produce the risky asset Y. Denote by $\Pi^h = z_x + p z_y$ the profits associated to production plan (z_x, z_y).

Agents get no utility from holding the assets X and Y. The number of agents can be finite or infinite, and utilities $U^h(X, Y, x_U, x_D) = \gamma_U^h u^h(x_U) + \gamma_D^h u^h(x_D)$ allow for different attitudes toward risk in terminal consumption. Finally, each agent $h \in H$ has an endowment x_{0^*} of X at time 0, and no other endowment.[7]

Finally we define the C-model as a special case of the C^*-model where there is a continuum of agents $h \in H = [0, 1]$.[8] Each agent is risk neutral with subjective probabilities, $(\gamma_U^h, \gamma_D^h = 1 - \gamma_U^h)$. The expected utility to agent h is $U^h(X, Y, x_U, x_D) = \gamma_U^h x_U + \gamma_D^h x_D$. We assume that γ_U^h is strictly increasing and continuous in h. If $\gamma_U^h > \gamma_U^{h'}$, we shall say that agent h is more optimistic (about state U) than agent h'.

The set J of contracts is defined in the next section.

II. Investment and Welfare in C and C^*-Models

In this section, we present our propositions regarding investment and welfare in C and C^*-models. In Section V, we analyze the equilibria corresponding to different financial innovations in detail and provide proofs of the propositions.

A. *Financial Innovation and Collateral*

A vitally important source of financial innovation involves the possibility of using assets and firms as collateral to back promises. Financial innovation in our model is described by a different set J. We shall always write $J = J^X \cup J^Y$, where J^X is the set of contracts backed by one unit of X and J^Y is the set of contracts backed by one unit of Y.

In Section IIA, we describe four different J regimes, which together with our description of the C and C^*-models in Section IH completes the specification of our model under four financial innovations. We also provide a brief description of equilibrium in the C-model for each of the regimes. In Section IIB, we present numerical simulations of equilibrium in the four regimes to illustrate the propositions we present in Sections IIC and IID.

N-Economy in the C-Model.—In the N-economy no financial contracts can be traded using the assets as collateral, and hence $J = \emptyset$. This situation corresponds to financial autarky. Since $Z_0^h = Z_0$ is convex, without loss of generality we may suppose that every agent chooses the same production plan (z_x, z_y) and $\Pi^h = \Pi$. Since we have normalized the mass of agents to be 1, (z_x, z_y) is also the aggregate production. In equilibrium there is a marginal buyer h_1.[9] All agents $h > h_1$ use all their endowment and profits from production $x_{0^*} + \Pi$ and buy all the Y produced

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[7] The assumption that agents have no endowment of Y is not needed for the five propositions on investment and welfare in Section II, but it is required for the nonexistence result in Section III.

[8] We suppose that agents are uniformly distributed in $[0, 1]$, that is they are described by Lebesgue measure.

[9] This is because of the linear utilities, the continuity of utility in h and the connectedness of the set of agents H at state $s = 0$.
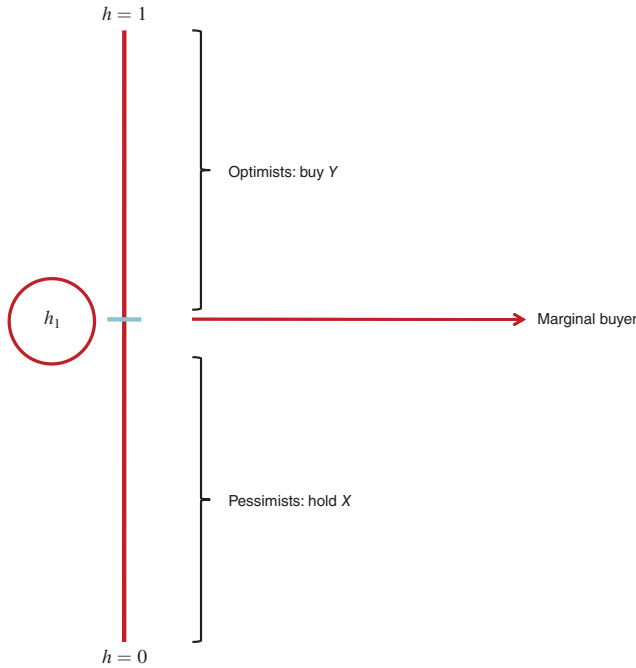
FIGURE 4. EQUILIBRIUM REGIME IN THE N-ECONOMY FOR THE C-MODEL

in the economy. Agents $h < h_1$ instead hold all the remaining X in the economy. Figure 4 describes the equilibrium regime in the N-economy for the C-model.

L-Economy in the C-Model.—The first type of financial innovation we consider is leverage. Agents can issue noncontingent promises (debt) of the consumption good using the risky asset as collateral. In this case $J = J^Y$, and each contract j uses one unit of asset Y as collateral and promises (j, j) units of consumption in the two states U, D, for all $j \in J = J^Y$. We call this the L-economy. We assume $j^* = d_D^Y \in J$, but otherwise the range of riskless promises $(j, j) \in J$ can be arbitrarily big or small.

Let us briefly describe the equilibrium. As before, (z_x, z_y) is also the aggregate production. Geanakoplos (2003) and Fostel and Geanakoplos (2012a) showed that in C-models, the only contract actively traded in equilibrium is $j^* = d_D^Y$ and that the riskless interest rate is zero. Agents have the choice of leveraging at any level they like, but leverage is endogenously determined in equilibrium in C-models at the loan to value $LTV = d_D^Y/p$. Borrowers are constrained: if they wish to borrow more on the same collateral by selling $j > j^*$, they would have to promise sharply higher interest j/π_j.

In equilibrium, there is a marginal buyer h_1 at state $s = 0$ whose valuation $\gamma_U^{h_1} d_U^Y + \gamma_D^{h_1} d_D^Y$ of the risky asset Y is equal to its price p. The optimistic agents $h > h_1$ collectively buy all the risky asset z_y produced in the economy, financing or leveraging their purchases with debt, that is, they buy Y and sell the riskless contract j^*, at a price of π_{j^*}, using the asset as collateral. In doing so, they are effectively buying the Arrow security that pays in the U state (since at D, their net payoff after debt repayment is 0). The pessimistic agents $h < h_1$ buy all the remaining safe assets
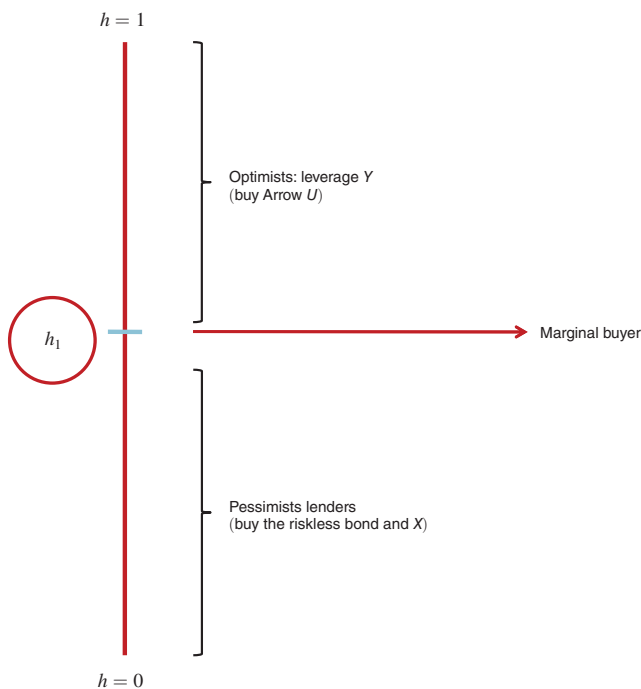
FIGURE 5. EQUILIBRIUM REGIME IN THE L-ECONOMY FOR THE C-MODEL

and lend to the optimistic agents by buying the riskless bond. Figure 5 shows the equilibrium regime in the L-economy for the C-model.

CDS-Economy in the C-Model.—The second type of financial innovation we consider is a Credit Default Swap on the risky asset Y. A Credit Default Swap (CDS) on the asset Y is a contract that promises to pay 0 when Y pays d_U^Y, and promises $d_U^Y - d_D^Y$ when Y pays only d_D^Y. CDS is a derivative, since its payoffs depend on the payoff of the underlying asset Y. A seller of a CDS must post collateral, typically in the form of money. In a two-period model, buyers of the CDS would insist on at least $d_U^Y - d_D^Y$ units of X as collateral. Thus, for every one unit of payment, one unit of X must be posted as collateral. We can therefore incorporate CDS into the leverage economy by taking J^X to consist of one contract promising $(0, 1)$.[10]

Thus, we take $J = J^X \cup J^Y$ where J^X consists of a contract promising $(0, 1)$, and J^Y consists of contracts (j, j) as described in the L-economy above. We call this the *CDS*-economy. Selling a CDS using X as collateral is like "tranching" the riskless asset into Arrow securities. The holder of X can get the Arrow U security by selling the CDS using X as collateral. The buyer of a CDS is effectively buying the Arrow D security.

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[10] A CDS can be "covered" or "naked" depending on whether the buyer of the CDS needs to hold the underlying asset Y. Notice that holding the asset and buying a CDS is equivalent to holding the riskless bond, which was already available without CDS in the L-economy, and hence the equilibrium would remain unchanged. For this reason in what follows we will focus on the case of naked CDS.

Strictly speaking, since the CDS is a derivative based on Y, if Y ceases to exist because it is not produced, then the CDS should disappear, as we make clear in Section III. For the rest of Section II we shall ignore this problem and assume that the CDS contract is traded whether or not Y is produced.

As before, we may suppose that every agent chooses the same production plan (z_x, z_y) and $\Pi^h = \Pi$, and thus (z_x, z_y) is also the aggregate production. The equilibrium, however, is more subtle in this case. There are two marginal buyers $h_1 > h_2$. Optimistic agents $h > h_1$ hold all the X and all the Y produced in the economy, selling the bond $j^* = d_D^Y$ at a price of π_{j^*}, using Y as collateral, and selling CDS at a price of π_C, using X as collateral. Hence, they are effectively buying the Arrow U security (the payoff net of debt and CDS payment at state D is zero). Moderate agents $h_2 < h < h_1$ buy the riskless bonds sold by more optimistic agents, earning a strictly positive rate of interest. Finally, agents $h < h_2$ buy the CDS from the most optimistic investors (so they are effectively buying the Arrow D). The equilibrium in the *CDS*-economy for the *C*-model is described in Figure 6.

In the remainder of this section we will compare the equilibrium prices, investment and welfare across these economies and present our main results. In Section V we will delve into the details of how these different equilibria are characterized and provide geometrical proofs for all the results that follow.

Arrow-Debreu in the C-Model.—Collateral equilibrium can implement the Arrow-Debreu equilibrium. Consider the economy defined by the set of available financial contracts as follows. We take $J = J^X \cup J^Y$ where J^X consists of a single contract promising $(0, 1)$ and J^Y consists of a single contract $(0, d_D^Y)$. In this case both assets in the economy can be used as collateral to issue the Arrow D promise, that is, both assets X and Y can be perfectly tranched into Arrow securities. Since there are no endowments in the terminal states, all the cash flows in the economy get tranched into Arrow U and D securities, and hence the collateral equilibrium in this economy is equivalent to the Arrow-Debreu equilibrium.

In the Arrow-Debreu equilibrium there is a marginal buyer h_1. All agents $h > h_1$ use all their endowment and profits from production $x_{0^*} + \Pi$ and buy all the Arrow U securities in the economy. Agents $h < h_1$ instead buy all the Arrow D securities in the economy. Figure 7 describes the equilibrium regime in Arrow-Debreu economy for the *C*-model.

B. *Numerical Examples*

We present numerical examples in order to motivate the propositions that follow. Consider a *C*-model with constant returns to scale technology $Z_0 = \{z = (z_x, z_y) \in R_- \times R_+ : z_y = -kz_x\}$, where $k \geq 0$. Beliefs are given by $\gamma_U^h = 1 - (1 - h)^2$ for $h \in (0, 1)$, and parameter values are $x_{0^*} = 1$, $d_U^Y = 1$, $d_D^Y = 0.2$ and $k = 1.5$. Table 1 presents the equilibrium in the four economies we just described. Figure 8 shows total investment in Y, $-z_x$, in each economy for different values of k.[11]
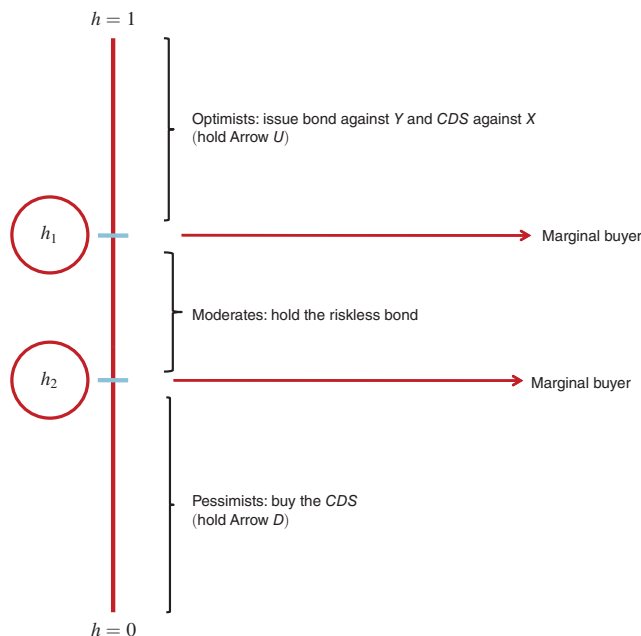
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[11] Appendix A presents numerical values.

Figure 6. Equilibrium Regime in the CDS-Economy for the C-Model

Investment is highest in the *L*-economy and is lowest in the *CDS*-economy. The most important lesson coming from this numerical example is that financial innovation affects investment decisions, even without any change in fundamentals. Notice that across the four economies we do not change fundamentals such as asset payoffs or productivity parameters, utilities or endowments. The only variation is in the type of financial contracts available for trade using the assets as collateral, as described by the different sets *J*. In other words, financial innovation drives investment variations. We formalize these results in Sections IIC, IID, and V.

It is also interesting to study the welfare implications of these financial innovations. Figure 9 shows the welfare corresponding to tail agents as well as the different equilibrium marginal buyers in each economy (calculated based on individual beliefs) when $k = 1.5$, across the four different economies.[12] The Arrow-Debreu equilibrium Pareto dominates the *L*-economy equilibrium, which in turn dominates the *N*-economy. However, no such domination holds for the *CDS*-economy. In particular, moderate agents are better off in the *CDS*-economy than in Arrow-Debreu. We will formally discuss these results in Sections IIC and V.

C. *Leverage, Overinvestment, and Welfare*

First we show that when agents can leverage the risky asset in the *L*-economy, price and investment levels are above those of the *N*-economy and Arrow-Debreu

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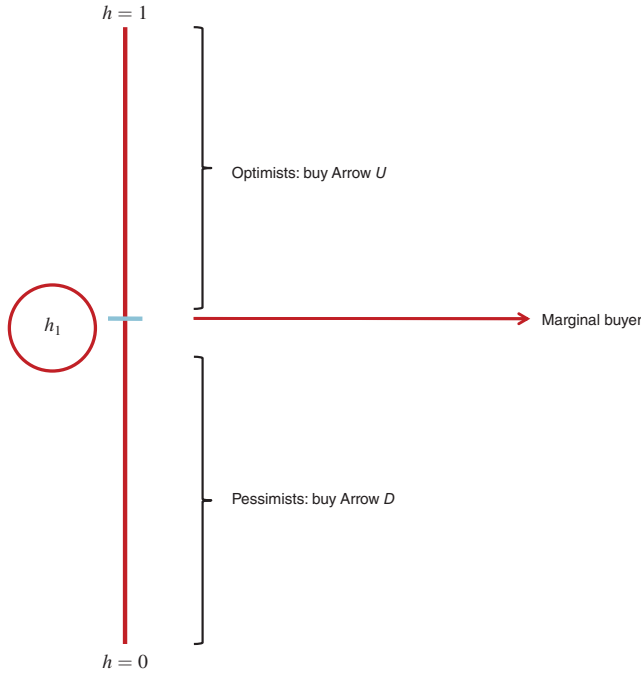[12] Appendix A presents numerical values.

FIGURE 7. EQUILIBRIUM REGIME IN THE ARROW-DEBREU ECONOMY FOR THE C-MODEL

TABLE 1—EQUILIBRIUM FOR $k = 1.5$

L-economy		N-economy		Arrow-Debreu economy		CDS-economy	
p	0.6667	p	0.6667	q_Y	0.6667	p	0.6667
h_1	0.3545	h_1	0.3545	q_U	0.5833	π_{j^*}	0.1904
z_x	−0.92	z_x	−0.6454	q_D	0.4167	π_C	0.4046
z_y	1.38	z_y	0.9682	h_1	0.3545	h_1	0.3880
				z_x	−0.2131	h_2	0.3480
				z_y	0.3197	z_x	−0.14
						z_y	0.2

economy. Hence, leverage generates over valuation and over investment with respect to both financial autarky and the first best. Our numerical example is consistent with a general property of the C-model as the following proposition shows.

PROPOSITION 1: *Leverage and Overinvestment in C-Models.*

Let $\left(p^L, \left(z_x^L, z_y^L\right)\right)$, $\left(p^A, \left(z_x^A, z_y^A\right)\right)$, and $\left(p^N, \left(z_x^N, z_y^N\right)\right)$ *denote the asset price and aggregate outputs for any equilibria in the L-economy, Arrow-Debreu economy, and the N-economy, respectively. Then:* (i) $\left(p^L, z_y^L\right) \geq \left(p^A, z_y^A\right)$ *and at least one of the two inequalities is strict, except possibly when* $z_x^L = -x_{0^*}$ *or* $z_x^L = 0$, *in which cases all that can be said is that* $z_y^L \geq z_y^A$. (ii) $\left(p^L, z_y^L\right) \geq \left(p^N, z_y^N\right)$ *and at least one of the two inequalities is strict, except possibly when* $z_x^L = -x_{0^*}$ *or* $z_x^L = 0$, *in which cases all that can be said is that* $z_y^L \geq z_y^N$.
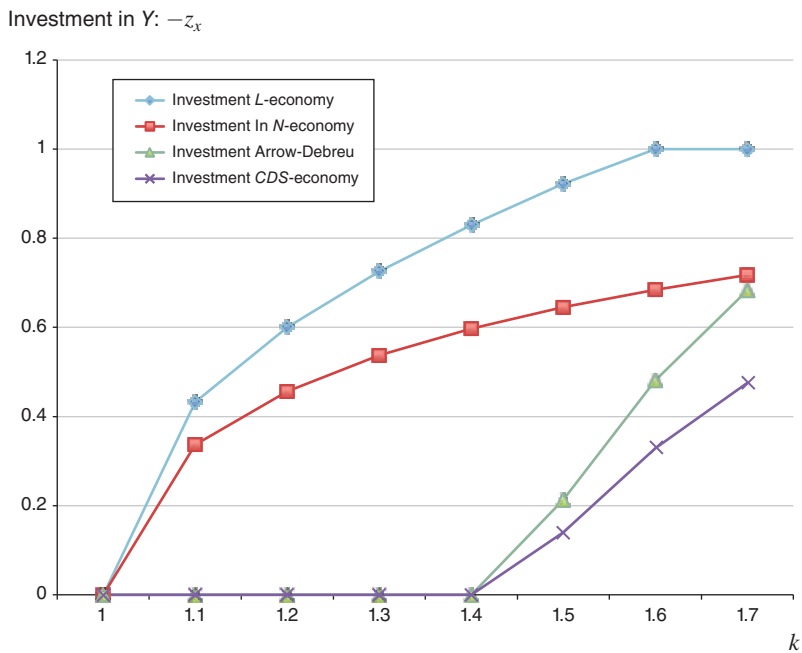
Investment in Y: $-z_x$

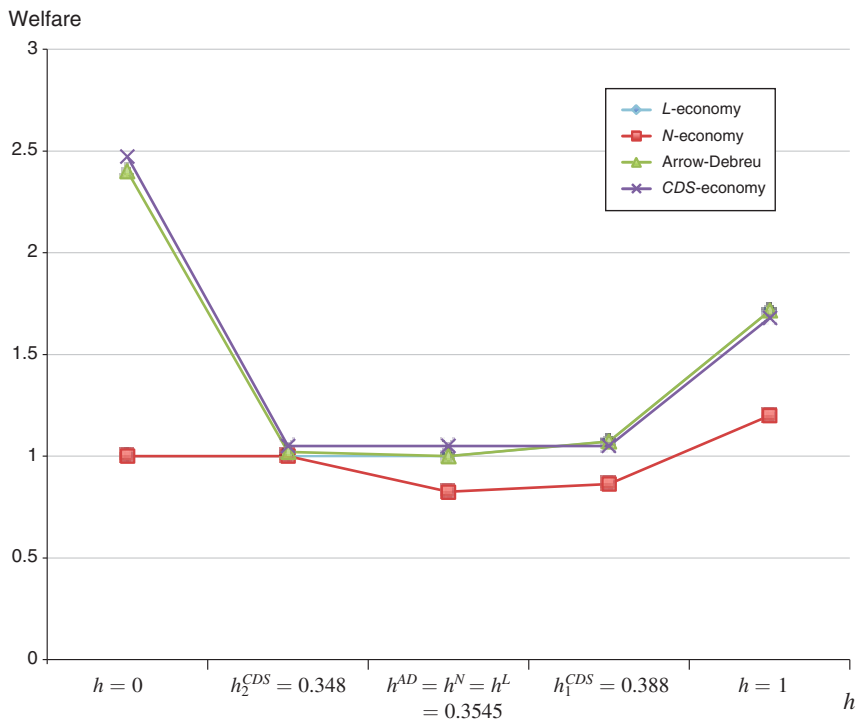FIGURE 8. TOTAL INVESTMENT IN Y, $-z_x$, IN DIFFERENT ECONOMIES FOR VARYING k

Welfare

FIGURE 9. FINANCIAL INNOVATION AND WELFARE

PROOF:

See Section VC.

One way to understand this result is in terms of collateral value as in Fostel and Geanakoplos (2008).[13] In the L-economy the risky asset can be used as collateral to issue debt. This gives the risky asset an additional collateral value. Consider the numerical example from Section IIB (see Table 1) and the optimistic agent $h = 0.9$. If she buys Y with cash she gets $\frac{\gamma_U(0.9)d_U^Y + (1 - \gamma_U(0.9))d_D^Y}{p} = \frac{0.99(1) + 0.01(0.2)}{0.67} = 1.48$ utiles per dollar invested. If she buys Y using leverage she gets $\frac{\gamma_U(0.9)(d_U^Y - d_D^Y)}{p - \pi_{j^*}} = \frac{0.99(1 - 0.2)}{0.67 - 0.2} = 1.70$ utiles per dollar of downpayment. The utility 1.48 per dollar from holding Y for its dividends alone is less than the utility 1.70 per dollar derived from leveraging Y, which is the best use of agent $h = 0.9$'s money. Therefore, $\mu^{h=0.9} = 1.70$ is the marginal utility of a dollar for agent $h = 0.9$.

Y has a collateral value beyond its payoff value. We can measure these in dollar equivalents as follows. The *Payoff Value* of Y for agent $h = 0.9$ is given by the marginal utility of Y measured in dollar equivalents, or $PV_Y^{h=0.9} = \frac{0.99(1) + 0.01(0.2)}{\mu^{h=0.9}} = 0.59 < p$. The *Collateral Value* of Y for agent $h = 0.9$ is given by the extra utility she can get by borrowing on Y, measured in dollar equivalents, $CV_Y^{h=0.9} = \frac{0.2(1.7-1)}{1.7} = 0.08 = 0.67 - 0.59 = p - PV_Y$. On the other hand, X cannot be used as collateral, so $PV_X = 1$ and $CV_X = 0$. Agents have more incentive to produce goods that are better collateral as measured by their collateral values. Investment migrates to better collateral.

Another (and related) way to understand the result is to note that leverage effectively tranches the Y payoff $z_y(1, 0.2)$ into $z_y(1 - 0.2) = 1.1$ Arrow U securities, held by optimists $h \in (0.35, 1)$, paying $z_y(1 - 0.2, 0)$, and $z_y(0.2) = 0.27$ riskless bonds, held by $h \in (0, 0.035)$ paying $z_y(0.2, 0.2)$. By dividing up the risky asset payoffs into two different kinds of assets, attractive to two different clienteles, demand is increased, and hence agents have more incentive to produce Y.

It turns out that Proposition 1 is valid for any type of preferences or space of agents as the following proposition shows.

PROPOSITION 2: *Leverage and Overinvestment in C*-Models.*

Let $\left(p^L, \left(z_x^L, z_y^L\right)\right)$ denote an equilibrium in the L-economy in which not all of x_{0^*} is used up in production. Suppose also that there is no other equilibrium in the L-economy with the same prices and more output. Then there exist equilibria $\left(p^A, \left(z_x^A, z_y^A\right)\right)$ and $\left(p^N, \left(z_x^N, z_y^N\right)\right)$ for the Arrow-Debreu economy and the N-economy respectively in which $\left(p^L, z_y^L\right) \geq \left(p^A, z_y^A\right)$ and $\left(p^L, z_y^L\right) \geq \left(p^N, z_y^N\right)$.

PROOF:

See Section VC.

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[13] We formally discuss these concepts in detail in Section IV.

We cannot compare production in the Arrow-Debreu economy with production in the N-economy. In the numerical example, production is higher in the N-economy, but with different parameters the reverse could be true.

Finally, the following welfare comparison holds when there are constant returns to scale in production.

PROPOSITION 3: *Welfare comparison with Constant Return to Scale in C*-Models.*

Let $\left(p^L, \left(z_x^L, z_y^L\right)\right)$, $\left(p^N, \left(z_x^N, z_y^N\right)\right)$ and $\left(p^A, \left(z_x^A, z_y^A\right)\right)$ *denote the asset price and aggregate outputs for any interior equilibria in the L-economy, N-economy and the Arrow-Debreu economy, respectively.*[14] *Suppose that Z_0 exhibits constant returns to scale. Then the Arrow-Debreu equilibrium Pareto-dominates the L-equilibrium, which in turn Pareto-dominates the N-equilibrium.*

PROOF:
See Section VC.

D. *CDS and Underinvestment*

We show that introducing a CDS on Y using X as collateral, in addition to leverage Y, generates undervaluation and underinvestment with respect to the investment level in the L and Arrow-Debreu economies. The result coming out of our numerical example is a general property of our C-model as the following propositions show.

PROPOSITION 4: *CDS underinvestment compared to Leverage in C-Models.*

Let $\left(p^L, \left(z_x^L, z_y^L\right)\right)$ and $\left(p^{CDS}, \left(z_x^{CDS}, z_y^{CDS}\right)\right)$ *denote the asset price and aggregate outputs for any equilibria in the L-economy and the CDS-economy, respectively. Then, $\left(p^L, z_y^L\right) \geq \left(p^{CDS}, z_y^{CDS}\right)$ and at least one of the two inequalities is strict, except possibly when $z_x^L = -x_{0*}$ or $z_x^L = 0$, in which case all that can be said is that $z_y^L \geq z_y^{CDS}$.*

PROOF:
See Section VE.

The basic intuition is along the same lines discussed after Proposition 1. Notice that selling a CDS using X as collateral is like "tranching" the riskless asset into Arrow securities. The holder of X can get the Arrow U security by selling the CDS using X as collateral. Hence, in the *CDS*-economy, the Arrow U security can be created through both X and Y, whereas in the *L*-economy only thorough Y. This gives less incentive in the *CDS*-economy to invest in Y.

The intuition can also be seen in terms of the collateral values of the input X and the output Y. Using the same numerical example as before, the marginal utility of

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[14] *Equilibrium is interior when $-x_{0*} < z_x < 0$, i.e when Y is positively produced and positive X remains.*

money at time 0 for $h = 0.9$ is given by $\mu^{h=0.9} = \frac{\gamma_U(0.9)\left(d_U^Y - d_D^Y\right)}{p - \pi_{j^*}} = \frac{0.99(1 - 0.2)}{0.67 - 0.1904}$
$= \frac{\gamma_U(0.9)\left(d_U^X\right)}{1 - \pi_C} = \frac{0.99(1)}{1 - 0.40} = 1.66$ (optimists in the *CDS*-economy buy the Arrow U
security using either X as collateral to sell *CDS* or Y as collateral to sell the
riskless bond). The payoff value of Y for agent $h = 0.9$ is given by $PV_Y^{h=0.9}$
$= \frac{0.99(1) + 0.01(0.2)}{\mu^{h=0.9}} = 0.60 < p$ and therefore the collateral value of Y for
agent $h = 0.9$ is given by $CV_Y^{h=0.9} = p - PV_Y^{h=0.9} = 0.67 - 0.6 = 0.07$. In the
CDS-economy X can also be used as collateral. The payoff value of X for agent
$h = 0.9$ is given by $PV_X^{h=0.9} = \frac{0.99(1) + 0.01(1)}{\mu^{h=0.9}} = 0.60$ and therefore the
collateral value of X for agent $h = 0.9$ is given by $CV_X^{h=0.9} = 1 - P$
$V_X^{h=0.9} = 1 - 0.60 = 0.40$. So whereas the collateral value of Y accounts for 10.5
percent of its price, the collateral value of X accounts for 40 percent of its price.

In our numerical example the price of Y is the same across the different econo-
mies (given the constant return to scale technology), but financial innovation affects
the collateral value of assets. Leverage increases the collateral value of Y relative to
X and *CDS* has the opposite effect. Investment responds to these changes in collat-
eral values, migrating to those assets with higher collateral values.

Finally, investment in the *CDS*-economy falls even below the investment level in
the Arrow-Debreu economy, provided that we make the additional assumption that
$\gamma_U(h)$ is concave. This assumption on beliefs can be interpreted as there is more
heterogeneity in beliefs among the pessimists than among the optimists.

PROPOSITION 5: *CDS underinvestment compared to Arrow-Debreu in C-Models.*

Let $\left(p^A, \left(z_x^A, z_y^A\right)\right)$ and $\left(p^{CDS}, \left(z_x^{CDS}, z_y^{CDS}\right)\right)$ denote the asset price and aggregate
outputs for any equilibria in the Arrow-Debreu economy and the CDS-economy,
respectively. Suppose $\gamma_U(h)$ is concave in h. Then, $\left(p^A, z_y^A\right) \geq \left(p^{CDS}, z_y^{CDS}\right)$ and
at least one of the two inequalities is strict, except possibly when $z_x^{CDS} = -x_0^*$,
or $z_x^{CDS} = 0$, in which case all that can be said is that $z_y^A \geq z_y^{CDS}$.

PROOF:

See Sections III and VE.

Propositions 4 and 5 cannot be generalized to the C^*-models, neither can we prove
unambiguous welfare results. The reason is that in the L-economy, N-economy, and
Arrow-Debreu economy there are state prices that can be used to value every asset
and contract.[15] In the *CDS*-economy this is not the case. We further discuss this in
Section V.

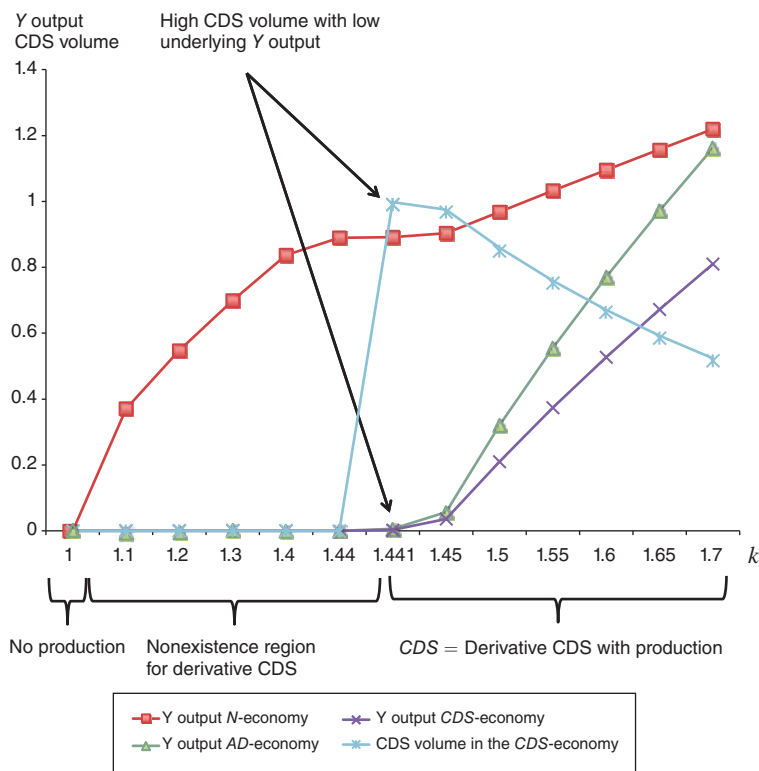[15] For details see Fostel and Geanakoplos (forthcoming).

FIGURE 10. CDS AND NONEXISTENCE

III. CDS and Nonexistence

By definition, a derivative contract does not trade when the underlying asset does not exist. It is precisely this fact that can bring about interesting nonexistence properties as we now show.

Let us define a Derivative CDS equilibrium as equal to the equilibrium in the *CDS*-economy defined in Section II if *Y* is produced, and as equal to the equilibrium in the *N*-economy if *Y* is not produced, since in that case neither the leverage debt nor the *CDS* trade. Thus, if all *CDS*-equilibria involve no production of *Y* and all *N*-equilibria involve positive production of *Y*, then there cannot exist a Derivative *CDS*-equilibrium.

Recall our numerical example in Section IIB. Observe that for all k such that $k \in (1, 1.44)$, the *N*-economy has positive production whereas the *CDS*-economy has no production. For that entire range, Derivatives *CDS*-equilibrium does not exist, as shown in Figure 10.[16]

CDS is a derivative, whose payoff depends on some underlying instrument. The quantity of CDS that can be traded is not limited by the market size of the underlying instrument. If the stock of the underlying security diminishes, the CDS trading may

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[16] Numerical values are in Appendix A. We could also find an example of nonexistence in economies with convex technologies, provided that Inada conditions (which prevent equilibrium production to be zero) are assumed away.

continue at the same high levels, as shown in the figure as k diminishes towards 1.44. But when the stock of the underlying instrument falls to zero, CDS trading must come to an end by definition. This discontinuity can cause robust nonexistence. The classical nonexistence observed in Hart (1975), Radner (1979), and Polemarchakis and Ku (1990) stemmed from the possibility that asset trades might tend to infinity when the payoffs of the assets tended toward collinear. A discontinuity arose when they became actually collinear. Collateral restores existence by (endogenously) bounding the asset trades. In our model, CDS trades stay bounded away from zero and infinity even as production disappears. Collateral does not affect this, since the bounded promises can be covered by the same collateral. But the moment production disappears, the discontinuity arises, since then CDS sales must become zero.

IV. No Marginal Underinvestment

Repayment enforceability problems restrict borrowing and thus naturally raise the specter of underinvestment. But when a commodity can serve as collateral, it thereby acquires an additional usefulness, and an opposite force is created which tends to overvaluation and overproduction of the commodity. In this section we show that under the general conditions of the model in Section I, at the margin, the overproduction force always dominates, despite the fact that agents are constrained in what they can borrow.

It is commonly said that when agents are borrowing constrained, they will be unable to invest as much as they would choose in a first-best world. The requirement to use scarce collateral in order to borrow is thus commonly supposed to explain low investment. However, Proposition 6 shows that when agents are collateral constrained, consumption is always more urgent than investment in collateral goods. If they were suddenly given a little extra money to make purely cash purchases, none of them would choose to buy or produce more of any good that can be used as collateral. Thus, on the margin, collateral requirements explain overinvestment rather than underinvestment. Of course if they were given enough money, or allowed to borrow a very large amount from the future, as in a first-best world, then they might well invest. Although there is never underinvestment on the margin in collateral equilibrium, investment could be above or below the first-best Arrow-Debreu level. In C and C^* economies there is always overinvestment with respect to the Arrow-Debreu level. But there are other economies in which there is underinvestment relative to the first-best Arrow-Debreu levels. (See Geanakoplos 1997 or Kiyotaki and Moore 1997).

The intuition that collateral requirements limit borrowing and thus lead to underinvestment seems even more compelling when agents have heterogeneous productivities. However, Proposition 7 shows that in the absence of cash flow problems, consumption is still more urgent than even the best investments. When an agent is collateral constrained, she would never spend a newfound dollar to buy or produce more of any good that can be used as collateral, even if she had access to the best technology in the economy for that investment. This result stands in contrast to the traditional macroeconomic literature on financial frictions in which cash flow problems are commonly assumed. For example, in Kiyotaki and Moore (1997),

gatherers end up holding land on which they are not productive when farmers could have produced more with it. This underinvestment arises from their cash flow problem assumption—that fruit growing on the land cannot be confiscated along with the land in case of default. This additional cash flow problem, and not the requirement for collateral, prevents farmers from borrowing enough to buy more land.

Propositions 6 and 7 show that collateral without a cash flow problem tends to create overproduction, not underproduction, on the margin. But they also point to a way of generating a large swing from overproduction to underproduction: move from a situation in which a good can be fully collateralized to one in which it can't be used as collateral at all.

We now make these ideas precise using the notions of collateral value and liquidity value from Fostel and Geanakoplos (2008) and Geanakoplos and Zame (2014). To simplify the statement of our marginal overinvestment propositions we shall assume differentiability of the utility functions for each agent.[17]

Given an equilibrium, let us define the marginal utility of money to agent h for any state $s \in S_T$ by

$$\mu_s^h = \frac{\partial u^h(\mathbf{x^h})}{\partial x_{s\ell}^h} \frac{1}{p_{s\ell}}$$

whenever $x_{s\ell}^h > 0$. Similarly, if 0ℓ is completely perishable, and $x_{0\ell}^h > 0$, define the marginal utility of money to agent h at 0 by

$$\mu_0^h = \frac{\partial u^h(x^{Idem})}{\partial x_{0\ell}^h} \frac{1}{p_{0\ell}}.$$

For possibly nonperishable commodities, define the *Payoff Value* for each commodity $\ell \in L_0$ to each agent h by

$$PV_{0\ell}^h = \frac{\dfrac{\partial u^h(\mathbf{x^h})}{\partial x_{0\ell}} + \displaystyle\sum_{s \in S_T} \mathbf{p}_s \cdot \mathbf{F}_s^h(\mathbf{1}_{0\ell}) \mu_s^h}{\mu_0^h},$$

and the *Collateral Value* for each commodity $\ell \in L_0$ to each agent h by

$$CV_{0\ell}^h = p_{0\ell} - PV_{0\ell}^h.$$

Similarly, we define the *Liquidity Value* of contract j to any (potential seller) h as

$$LV_j^h = \pi_j - \frac{\displaystyle\sum_{s \in S_T} \min\left(\mathbf{p}_s \cdot \mathbf{F}_s^h(\mathbf{c}_j), \mathbf{p}_s \cdot \mathbf{j}_s\right) \mu_s^h}{\mu_0^h}.$$

[17] More generally let us assume that every agent has strictly positive "extended" endowments of commodities in every state, where we define the extended endowment in state $s \in S_T$ to be $e_s^h + F_s^h(e_0^h)$. Given commodity and contract prices (p, π) define the indirect utility $U^h((p, \pi), w_0, w_1, \ldots, w_S)$ as the maximum utility agent h can get by trading at prices (p, π), where the $w_s \in (-\varepsilon, \varepsilon)$ represent small transfers of income, positive or negative. Since agents have strictly positive endowments, for small negative income transfers their starting endowment wealth will be positive in each state. Since utilities are concave, the indirect utility function must be concave in w, and hence differentiable from the right and the left at every point, including the point with equilibrium prices and $w = 0$. Let μ_s^h be the derivative from the right for states $s \in S_T$, and let μ_0^h be the derivative from the left for state $s = 0$.

Agent h is liquidity constrained in equilibrium if and only if there is some contract j, with $h \in H(j)$, that has strictly positive liquidity value to him. In equilibrium, we must have $LV_j^h \geq 0$ for all $h \in H, j \in J$, otherwise agent h ought to have bought more j.

Fostel and Geanakoplos (2008) and Geanakoplos and Zame (2014) proved that when $C_j = \mathbf{1}_{ol}$ and $\theta_j^h < 0$, $CV_{0\ell}^h = LV_j^h$, so that the liquidity value associated to any contract j that is actually issued using commodity ℓ as collateral equals the collateral value of the commodity.

The next proposition shows that there is never marginal underinvestment in goods that can be used as collateral.

PROPOSITION 6: *No Direct Marginal Underinvestment*

Consider a collateral equilibrium $\left((\mathbf{p}, \pi), (\mathbf{z}^h, \mathbf{x}^h, \boldsymbol{\theta}^h)_{h \in H} \right)$. Then, for every $h \in H, \ell \in L_0$, we must have $p_{0\ell} \geq PV_{0\ell}^h$.

Moreover, if there is some contract j, with $c_j = 1_{0\ell}$, that has strictly positive liquidity value to $h \in H(j)$, then $p_{0\ell} > PV_{0\ell}^h$. In this case, if h were given an extra unit of cash to make a purely cash purchase, she would not purchase or produce more of good 0ℓ.

PROOF:

If $p_{0\ell} < PV_{0\ell}^h$, then agent h ought to have reduced a little of what she was doing in equilibrium, and instead bought a little of commodity 0ℓ, a contradiction. If on top of buying a little 0ℓ, h could also use it to collateralize a little borrowing via contract j, with positive liquidity value, then we would also contradict $p_{0\ell} \leq PV_{0\ell}^h$. The concluding statement follows immediately. ∎

Next, we prove a stronger result: there is never marginal underinvestment even when agents are allowed to invest in technologies owned by other agents in the economy.

For each commodity $\ell \in L_0$, define its *Payoff Value to agent h' via agent h* by

$$PV_{0\ell}^{h,h'} = \frac{\dfrac{\partial u^h(\mathbf{x}^h)}{\partial x_{0\ell}}}{\mu_0^h} + \frac{\displaystyle\sum_{s \in S_T} \mathbf{p}_s \cdot \mathbf{F}_s^h(\mathbf{1}_{0\ell}) \mu_s^{h'}}{\mu_0^{h'}}.$$

PROPOSITION 7: *No Indirect Marginal Underinvestment*

Consider a collateral equilibrium $\left((\mathbf{p}, \pi), (\mathbf{z}^h, \mathbf{x}^h, \boldsymbol{\theta}^h)_{h \in H} \right)$. Suppose an agent h can use $1_{0\ell}$ as collateral, and in particular can issue a large contract j backed by $1_{0\ell}$. Then, for every $h' \in H, \ell \in L_0$, we must have $p_{0\ell} \geq PV_{0\ell}^{h,h'}$.

Moreover, if the large contract j has strictly positive liquidity value to $h \in H(j)$, then $p_{0\ell} > PV_{0\ell}^{h,h'}$. In this case, even if h' were given an extra unit of cash to make a purely cash purchases, she would not use it to buy 0ℓ or to pay h to use 0ℓ to produce dividends for her.

PROOF:

Agent h can always sell contract j to agent h' for at least $\pi_j \geq \dfrac{\sum\limits_{s \in S_T} \mathbf{p}_s \cdot \mathbf{F}_s^h (\mathbf{1}_{0\ell}) \mu_s^{h'}}{\mu_0^{h'}}$.

Hence, if $p_{0\ell} < PV_{0\ell}^{h, h'}$, agent h ought to have reduced a little of what he was doing in equilibrium, and instead bought a little of commodity 0ℓ, used it to issue the large contract j, thus, paying on net at most

$$p_{0\ell} - \pi_j < PV_{0\ell}^{h, h'} - \frac{\sum\limits_{s \in S_T} \mathbf{p}_s \cdot \mathbf{F}_s^h (\mathbf{1}_{0\ell}) \mu_s^{h'}}{\mu_0^{h'}} = \frac{\dfrac{\partial u^h(\mathbf{x^h})}{\partial x_{0\ell}}}{\mu_0^h},$$

and being better off, a contradiction. The rest follows as in Proposition 6. ∎

V. Equilibrium Analysis and Proofs

In this section, we fully characterize the equilibrium in the Arrow-Debreu, L, and CDS-economies for C-models presented in Section II.[18] We also use an Edgeworth box diagram to illustrate each case and to provide a geometrical proof of the results in Section II for C-models. We also prove much more briefly how some of these results can be extended to C^*-models.

A. *Arrow-Debreu Equilibrium*

Arrow-Debreu equilibrium in C and C^*-models is given by present value consumption prices (q_U, q_D), which without loss of generality we can normalize to add up to 1, and by consumption $(x_U^h, x_D^h)_{h \in H}$ and production $(z_x^h, z_y^h)_{h \in H}$ satisfying

(i) $\displaystyle\int_0^1 x_s^h \, dh = \int_0^1 \left(x_{0^*} + z_x^h + d_s^Y z_y^h \right) dh, \quad s = U, D.$ [19]

(ii) $(x_U^h, x_D^h) \in B_W^h (q_U, q_D, \Pi^h)$

$$\equiv \left\{ (x_U^h, x_D^h) \in R_+^2 : q_U x_U^h + q_D x_D^h \leq (q_U + q_D) x_{0^*} + \Pi^h \right\}.$$

(iii) $(x_U, x_D) \in B_W^h (q_U, q_D, \Pi^h) \Rightarrow U^h(x_U, x_D) \leq U^h(x_U^h, x_D^h), \forall h.$

(iv) $\Pi^h \equiv q_U \left(z_x^h + z_y^h d_U^Y \right) + q_D \left(z_x^h + z_y^h d_D^Y \right) \geq q_U \left(z_x + z_y d_U^Y \right) + q_D \left(z_x + z_y d_D^Y \right),$

$$\forall (z_x, z_y) \in Z_0^h.$$

Condition (i) says that supply equals demand for the consumption good at U and D. Conditions (ii) and (iii) state that each agent optimizes in her budget set, where income is the sum of the value of endowment x_{0^*} of X and the profit from her intra-period production. Condition (iv) says that each agent maximizes profits,

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[18] For brevity, the characterization of the equilibrium in the N-economy is given in Appendix C.
[19] We have used the integral to include the possibility of a continuum of agents, as in the C-model. The same equation with a summation sign applies when there is a finite number of agents.

where the price of X and Y are implicitly defined by state prices q_U and q_D as $q_X = q_U + q_D$ and $q_Y = q_U d_U^Y + q_D d_D^Y$.[20]

Specializing to C-models, in Arrow-Debreu equilibrium there is a marginal buyer h_1. All agents $h > h_1$ use all their endowment and profits from production $(q_U + q_D)x_{0^*} + \Pi = (x_{0^*} + \Pi)$ and buy all the Arrow U securities in the economy. Agents $h < h_1$ instead buy all the Arrow D securities in the economy.

It is clarifying to describe the equilibrium for C-models using the Edgeworth box diagram in Figure 11. The axes are defined by the potential total amounts of x_U and x_D available as dividends from the stock of assets emerging at the end of period 0. Point Q represents the economy total final output from the actual equilibrium choice of aggregate intra-period production (z_x, z_y), so $Q = (z_y d_U^Y + x_{0^*} + z_x, z_y d_D^Y + x_{0^*} + z_x)$, where we take the vertical axis U as the first coordinate.

The 45-degree dotted line in the diagram is the set of consumption vectors that are collinear with the dividends of the aggregate endowment x_{0^*}. The steeper dotted line includes all consumption vectors collinear with the dividends of Y. The curve connecting the two dotted lines is the aggregate intra-period production possibility frontier, describing how the aggregate endowment of the riskless asset, x_{0^*}, can be transformed into Y. As more and more X is transformed into Y, the total output in U and D gets closer and closer to the Y dotted line.

The equilibrium prices $q = (q_U, q_D)$ determine parallel price lines orthogonal to q. One of these price lines is tangent to the production possibility frontier at Q.

In the classical Edgeworth Box there is room for only two agents. One agent takes the origin as her origin, while the second agent looks at the diagram in reverse from the point of view of the aggregate point Q, because she will end up consuming what is left from the aggregate production after the first agent consumes. The question is, how to put a whole continuum of heterogeneous agents into the same diagram? When the agents have linear preferences and the heterogeneity is one-dimensional and monotonic, this can be done. Suppose we put the origin of agent $h = 0$ at Q. We can mark the aggregate endowment of all the agents between $h = 0$ and any arbitrary $h = h_1$ by its distance from Q. Since endowments are identical, and each agent makes the same profit, it is clear that this point will lie h_1 of the way on the straight line from Q to the origin at 0, namely at $(1 - h_1)Q = Q - h_1 Q$. The aggregate budget line of these agents is then simply the price line determined by q through their aggregate endowment, (their aggregate budget set is everything in the box between this line and Q). Of course looked at from the point of view of the origin at 0, the same point represents the aggregate endowment of the agents between $h = h_1$ and $h = 1$. (Since every agent has the same endowment, the fraction

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[20] These equations define any equilibrium in which some X remains after production. A more comprehensive definition of Arrow-Debreu equilibrium would explicitly include all four prices q_U, q_D, q_X, q_Y. In such an equilibrium $q_X \geq q_U + q_D$ and $q_Y \geq q_U d_U^Y + q_D d_D^Y$, otherwise any agent could sell the Arrow securities and buy the asset, owing nothing on net and pocketing the profit. As long as X, Y are both in positive supply after production, the reverse inequalities must hold, for otherwise the opposite trade would make a profit. If Y is not produced at all and $q_Y > q_U d_U^Y + q_D d_D^Y$, then we can lower q_Y until this becomes an equality without disturbing equilibrium, since then there would be even less incentive to produce. If all of X is used up in production, however, then we could be left with $q_X > q_U + q_D$, since nobody has X to sell, and lowering the price q_X might create more intended production. Thus the equations describing equilibrium in which no X remains after production consist of (i), (ii), (iii), and (iv), but where we weaken the inequality in (4) to hold only for $-x_{0^*} < z_x \leq 0$.
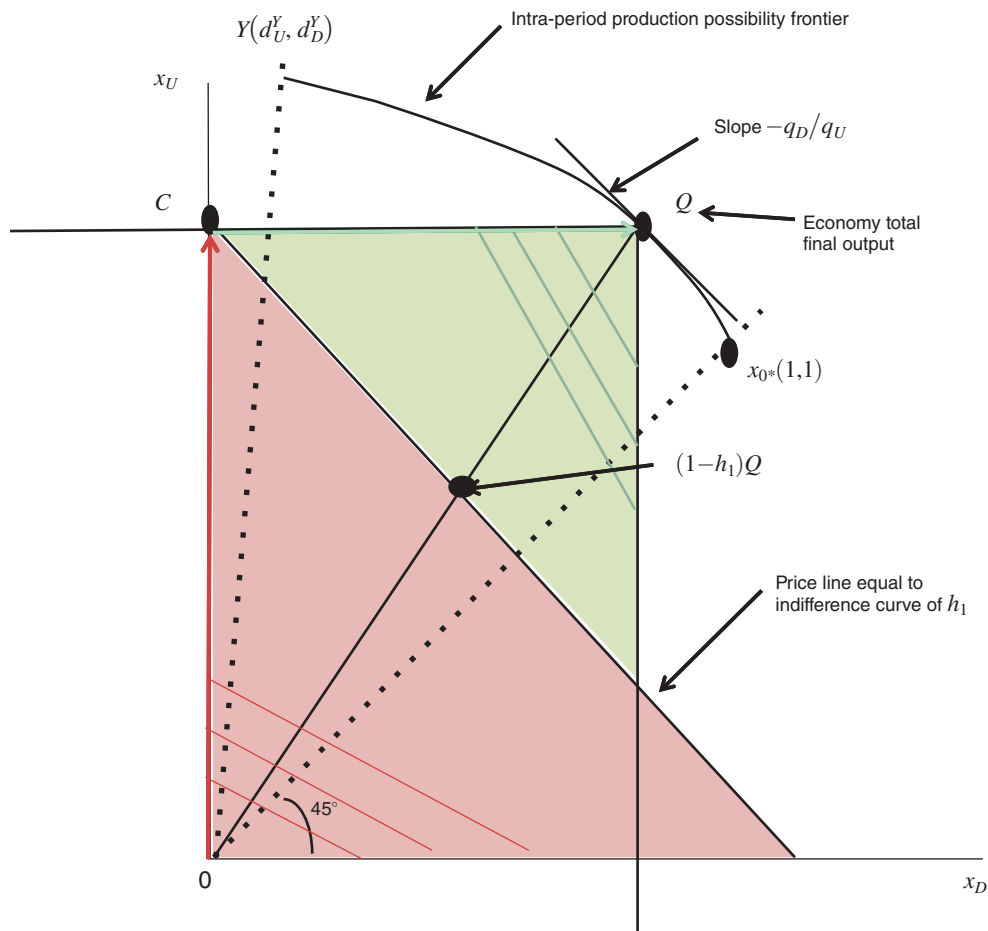
FIGURE 11. EQUILIBRIUM IN THE ARROW-DEBREU ECONOMY WITH PRODUCTION: EDGEWORTH BOX

$(1 - h_1)$ of the agents can afford to buy the fraction $(1 - h_1)$ of Q.) Therefore, the same price line represents the aggregate budget line of the agents between h_1 and 1, as seen from their origin at 0, (and their aggregate budget set is everything between the budget line and the origin 0).

At this point we invoke the assumption that all agents have linear utilities, and that they are monotonic in the probability assigned to the U state. Suppose the prices q are equal to the probabilities $\left(\gamma_U^{h_1}, \gamma_D^{h_1}\right)$ of agent h_1. Agents $h > h_1$, who are more optimistic than h_1, have flatter indifference curves, illustrated in the diagram by the indifference curves near the origin 0. Agents $h < h_1$, who are more pessimistic than h_1, have indifference curves that are steeper, as shown by the steep indifference curves near the origin Q. The agents more optimistic than h_1 collectively will buy at the point C, where the budget line crosses the x_U axis above the origin, consuming exclusively in state U. The pessimists $h < h_1$ will collectively choose to consume at the point where the budget line crosses the x_D axis through their origin at Q, the same point C, consuming exclusively in state D. Clearly, total consumption of optimists and pessimists equals Q, i.e., $\left(z_y d_U^Y + x_{0^*} + z_x, 0\right) + \left(0, z_y d_D^Y + x_{0^*} + z_x\right) = Q$.

From the previous analysis it is clear that the equilibrium marginal buyer h_1 must have two properties: (i) one of her indifference curves is tangent to the production possibility frontier at Q, and (ii) her indifference curve through the collective endowment point $(1 - h_1)Q$ cuts the top left point of the Edgeworth Box whose top right point is determined by Q.

Finally, the system of equations that characterizes the Arrow-Debreu equilibrium is given by

(3)
$$(z_x, z_y) \in Z_0$$

(4)
$$\Pi = z_x + q_Y z_y \geq \tilde{z}_x + q_Y \tilde{z}_y, \quad \forall (\tilde{z}_x, \tilde{z}_y) \in Z_0.$$

(5)
$$q_U d_U^Y + q_D d_D^Y = q_Y$$

(6)
$$\gamma_U^{h_1} = q_U$$

(7)
$$\gamma_D^{h_1} = q_D$$

(8)
$$(1 - h_1)(x_{0^*} + \Pi) = q_U \big((x_{0^*} + z_x) + z_y d_U^Y \big).$$

Equations (3) and (4) state that intra-production plans should be feasible and should maximize profits. Equation (5) uses state prices to price the risky asset Y. Equations (6) and (7) state that the price of the Arrow U and Arrow D are given by the marginal buyer's state probabilities. Equation (8) states that all the money spent on buying the total amount of Arrow U securities in the economy (described by the RHS) should equal the total income of the buyers (described by the LHS).[21]

B. *The L-Economy*

In the L-economy, $J = J^Y$, and each contract j uses one unit of asset Y as collateral and promises (j, j) for all $j \in J = J^Y$. Agents can issue debt using any contract; in particular they could choose to sell contract (d_U^Y, d_U^Y). But they do not. Geanakoplos (2003), and Fostel and Geanakoplos (2012a) proved that in the C-model, there is a unique equilibrium in which the *only* contract actively traded is $j^* = d_D^Y$ (provided that $j^* \in J$) and that the riskless interest rate equals zero.[22] Hence, $\pi_{j^*} = j^* = d_D^Y$ and there is no default in equilibrium. Even though agents are not restricted from selling bigger promises, the price π_j rises so slowly for $j > j^*$ that they choose not to issue $j > j^*$. In other words, they cannot borrow more on the same collateral without

[21] As mentioned in footnote 20, these equations apply to equilibria in which some X remains after production. For equilibria with $-z_x = x_{0^*}$, we need to weaken inequality (4) in exactly the same way we weakened the corresponding inequality at the beginning of this section. For such equilibria, Figure 11 changes so that Q is on the Y line, but the price line (indifference curve of h_1) might be flatter than the tangent to the production possibility frontier at Q. In case there is no production, the price line (indifference curve of h_1) might be steeper than another tangent to the production possibility frontier at Q, but it will always be a tangent.

[22] This holds only if there is a positive amount of X and Y in the economy after production. If there is no X remaining after production, then the interest rate can be positive. If there is no Y produced, then no loans are traded and the L-equilibrium is also an N-equilibrium.

raising the interest rate prohibitively fast: they are effectively constrained to j^*. Fostel and Geanakoplos (forthcoming) also showed that in every equilibrium in C and C^*-models there are unique state probabilities, such that X and Y and all the financial contracts are priced by their expected payoffs.[23]

As we saw in Section IIA, in equilibrium there is a marginal buyer h_1 at state $s = 0$ whose valuation $\gamma_U^{h_1} d_U^Y + \gamma_D^{h_1} d_D^Y$ of the risky asset Y is equal to its price p. The optimistic agents $h > h_1$ collectively buy all the risky asset z_y produced in the economy, financing this with debt contracts j^*. The pessimistic agents $h < h_1$ buy all the remaining safe assets and lend to the optimistic agents.

The endogenous variables to solve for are the price of the risky asset p, the marginal buyer h_1 and production plans (z_x, z_y). The system of equations that characterizes the equilibrium in the L-economy is given by

$$(9) \qquad\qquad\qquad (z_x, z_y) \in Z_0$$

$$(10) \qquad\qquad\qquad \Pi = z_x + p z_y \geq \tilde{z}_x + p \tilde{z}_y, \ \forall (\tilde{z}_x, \tilde{z}_y) \in Z_0.$$

$$(11) \qquad (1 - h_1)(x_{0^*} + \Pi) + d_D^Y z_y = p z_y$$

$$(12) \qquad\qquad \gamma_U^{h_1} d_U^Y + \gamma_D^{h_1} d_D^Y = p.$$

Equations (9) and (10) describe profit maximization. Equation (11) equates money $p z_y$ spent on the asset, with total income from optimistic buyers in equilibrium: all their endowment $(1 - h_1) x_{0^*}$ and profits from production $(1 - h_1) \Pi$, plus all they can borrow $d_D^Y z_y = \pi_{j^*} z_y$ from pessimists using the risky asset as collateral. Equation (12) states that the marginal buyer prices the asset.

We can also describe the equilibrium using the Edgeworth box diagram in Figure 12, provided that $-x_{0^*} < z_x < 0$. As in Figure 11, the axes are defined by the potential total amounts of x_U and x_D available as dividends from the stock of assets emerging at the end of period 0. The probabilities $\gamma^{h_1} = (\gamma_U^{h_1}, \gamma_D^{h_1})$ of the marginal buyer h_1 define state prices that are used to price x_U and x_D, and to determine the price lines orthogonal to γ^{h_1}. One of those price lines is tangent to the production possibility frontier at Q, representing the economy total final output, $Q = (z_y d_U^Y + x_{0^*} + z_x, z_y d_D^Y + x_{0^*} + z_x)$.

The dividend coming from the equilibrium choice of X, $x_{0^*} + z_x$, lies at the intersection of the "X-dotted" line starting from 0 and the "Y-dotted" line starting at Q. The dividends coming for the equilibrium investment in Y, $z_y (d_U^Y, d_D^Y)$, lies at the intersection of the "Y-dotted" line starting at 0 and the "X-dotted" line starting at Q.

Again, we put the origin of agent $h = 0$ at Q. We can mark the aggregate endowment of all the agents between 0 and any arbitrary h_1 by its distance from Q. Since endowments are identical, and each agent makes the same profit, it is clear that this

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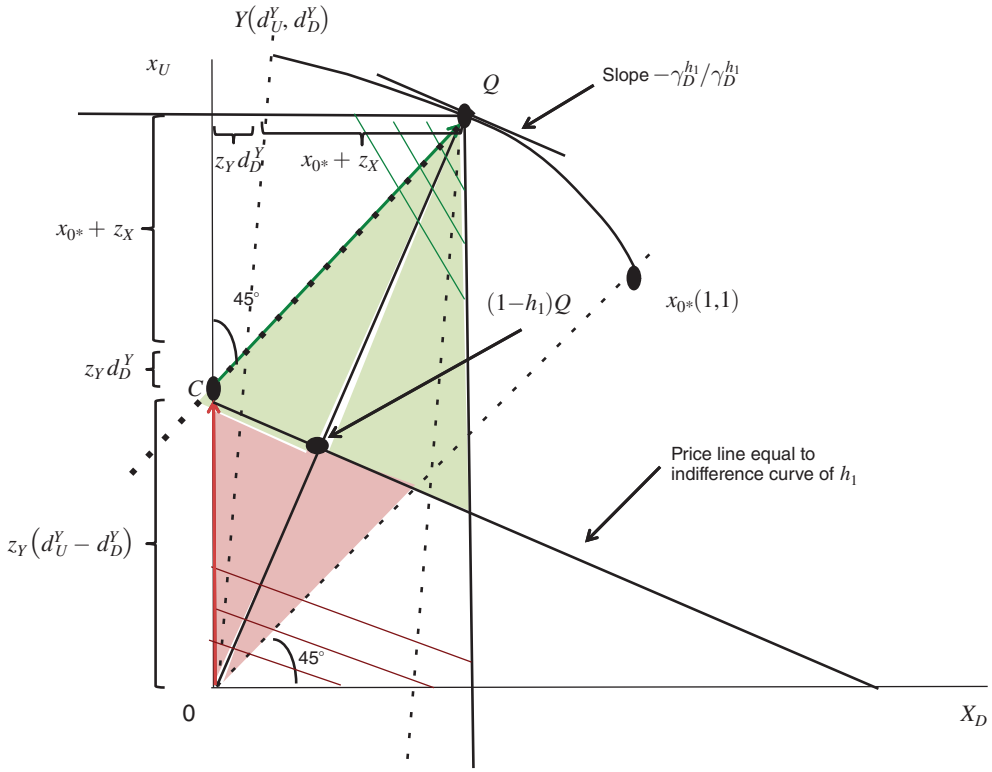[23] Again this is true as long as not all the X is used up in production.

FIGURE 12. EQUILIBRIUM REGIME IN THE *L*-ECONOMY: EDGEWORTH BOX

point will lie h_1 of the way on the line from Q to the origin, namely at $(1 - h_1)Q = Q - h_1 Q$. Similarly, the same point describes the aggregate endowment of all the optimistic agents $h > h_1$ looked at from the point of view of the origin at 0.

In equilibrium, optimists $h > h_1$ consume at point C. As in the Arrow-Debreu equilibrium they only consume in the U state. They consume the total amount of Arrow U securities available in the economy, $z_y(d_U^Y - d_D^Y)$. Notice that when agents leverage asset Y, they are effectively creating and buying a "synthetic" Arrow U security that pays $d_U^Y - d_D^Y$ and costs $p - d_D^Y$, namely at price $\gamma_U^{h_1} = (d_U^Y - d_D^Y)/(p - d_D^Y)$.

The total income of the pessimists between 0 and h_1 is equal to $h_1 Q$. Hence, looked at from the origin Q, the pessimists must also be consuming on the same budget line as the optimists. However, unlike the Arrow-Debreu economy, pessimists now must consume in the cone generated by the 45-degree line from Q and the vertical axis starting at Q. Since their indifference curves are steeper than the budget line, they will also choose consumption at C. However at C, unlike in the Arrow-Debreu equilibrium, they consume the same amount, $x_{0^*} + z_x + z_y d_D^Y$, in both states. Clearly, total consumption of optimists and pessimists equals Q, i.e., $\left(z_y(d_U^Y - d_D^Y), 0\right) + \left(x_{0^*} + z_x + z_y d_D^Y, x_{0^*} + z_x + z_y d_D^Y\right) = Q$.

From the previous analysis we deduce that the marginal buyer h_1 must satisfy two properties: (i) one of her indifference curves must be tangent to the production possibility frontier at Q, and (ii) her indifference curve through the point $(1 - h_1)Q$

must intersect the vertical axis at the level $z_y\left(d_U^Y - d_D^Y\right)$, which corresponds to point C and the total amount of Arrow U securities in equilibrium in the L-economy.[24]

C. Overinvestment and Welfare: Proofs

PROOF OF PROPOSITION 1:

Part (i): The Edgeworth Box diagrams in Figures 11 and 12 allow us to see why production is higher in the L-economy than in the Arrow-Debreu economy. In the L-economy, optimists collectively consume $z_y^L\left(d_U^Y - d_D^Y\right)$ in state $U,$ while in the Arrow-Debreu economy they consume $z_y^A d_U^Y + \left(x_0^* + z_x^A\right)$. The latter is evidently much bigger, at least as long as $z_y^A \geq z_y^L$. So let us begin with an interior L-equilibrium in Figure 11. Suppose, contrary to what we want to prove, that Arrow-Debreu output of Y were at least as high, $z_y^A \geq z_y^L$. Since the total economy output Q^L maximizes profits at the leverage equilibrium prices, at those leverage prices $\left(1 - h_1^L\right)Q^A$ is worth no more than $\left(1 - h_1^L\right)Q^L$. Thus, $\left(1 - h_1^L\right)Q^A$ must lie on the origin side of the h_1^L indifference curve through $\left(1 - h_1^L\right)Q^L$. Suppose also that the Arrow-Debreu price is higher than the leverage price: $p^A \geq p^L$. Then the Arrow-Debreu marginal buyer is at least as optimistic, $h_1^A \geq h_1^L$. Then, $\left(1 - h_1^A\right)Q^A$ would also lie on the origin side of the h_1^L indifference curve through $\left(1 - h_1^L\right)Q^L$. Moreover, the indifference curve of h_1^A would be flatter than the indifference curve of h_1^L and, hence, cut the vertical axis at a lower point. By property (ii) of the marginal buyer in both economies, this means that optimists would collectively consume no more in the Arrow-Debreu economy than they would in the leverage economy, a contradiction. It follows that either $z_y^A < z_y^L$ or $p^A < p^L$. But a routine algebraic argument from profit maximization (given in Appendix B) proves that if one of these strict inequalities holds, the other must also hold weakly in the same direction. (If the price of output is strictly higher, it cannot be optimal to produce strictly less.)[25]

Part (ii): The argument to prove the overinvestment with respect to financial autarky is completely analogous to the argument in part (i) using instead Figure 12 and Figure C1 (the Edgeworth box for the N-economy given in Appendix C). ∎

PROOF OF PROPOSITION 2:

At arbitrary state prices (p_U, p_D), define the budget sets of a fixed agent h in the three economies by Figure 13. The budget set for each agent h in the L-economy is equal to her budget set in the Arrow-Debreu economy restricted to the cone between the vertical axis and the 45-degree line, while in the N-economy the budget set is

[24] In case all X is used in production, the Figure can allow for the possibility in (i) that the indifference curve of the marginal buyer is flatter than the tangent of the production possibility frontier. In case there is no production, the marginal buyer must be $h_1 = 1$, and her indifference curve must be tangent to the production possibility frontier at $Q = x_{0^*}$. But this could be steeper than another tangent there that defines the relative prices of X and Y.

[25] If the L-equilibrium involves maximum production of Y, there is nothing to prove. Suppose the L-equilibrium has no production of Y, then it is an N-equilibrium and even agent $h = 1$ would not like to produce. We can then use exactly the same proof to contradict that there is positive production in Arrow-Debreu.
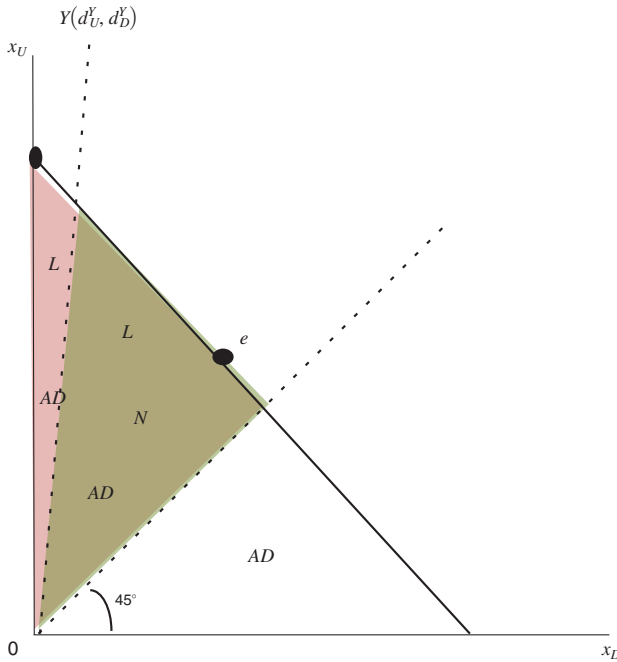
Figure 13. Budget Sets

further restricted to lie between the 45-degree line and the Y-payoff line. As shown in Fostel and Geanakoplos (forthcoming), so long as there is positive X remaining in the economy, the equilibrium asset and contract prices in all three economies N, L, and Arrow-Debreu are determined by the state prices (p_U, p_D), and the budget sets are as described.

Letting $p_U + p_D = 1$, we can use these budget sets to define the aggregate excess demand correspondence for x_U in each economy by $E_U^L(p_U)$, $E_U^{AD}(p_U)$, and $E_U^N(p_U)$, respectively. By strict monotonicity of the individual utilities, for all small enough p_U, every point in $E_U^{AD}(p_U)$ and $E_U^N(p_U)$ is positive.

Let there be an L-equilibrium with state prices (p_U^L, p_D^L) at which some X remains after production, and suppose there is no other L-equilibrium at those prices with more output Y. Then $0 \in E_U^L(p_U^L)$, and total demand must be equal to total supply $z_y^L d_U^Y$. Let (x_U^{hL}, x_D^{hL}) be the equilibrium consumption choice in the L-economy for each agent h. Suppose $z_y^L d_U^Y$ is not the maximum profit maximizing production at those prices. Then no agent is willing to demand more in the L-economy at those prices, for otherwise there would be an L-equilibrium with more production at those prices. Hence, by the nesting of the budget sets, any Arrow-Debreu equilibrium at those prices must involve the same or less demand and thus production. If there is no Arrow-Debreu equilibrium at those prices, every point in $E_U^{AD}(p_U^L)$ is negative. By the upper semi-continuity and convex valuedness of the correspondence E_U^{AD}, and the excess demand at low prices described in the last paragraph, there must be an Arrow-Debreu equilibrium state price $p_U < p_U^L$ with $0 \in E_U^{AD}(p_U)$. See Figure 14. But at a lower U state price, production cannot be higher. So some X must still
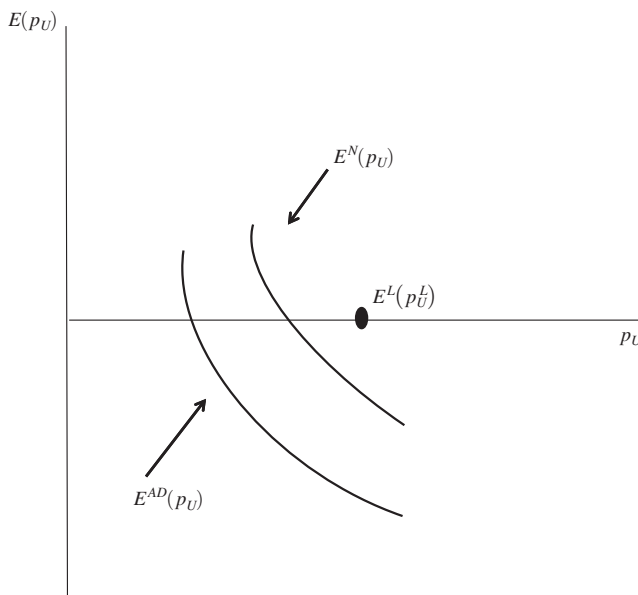
Figure 14. Excess Demands

remain, and we have a genuine Arrow-Debreu equilibrium. A similar argument works for N-economies. ∎

PROOF OF PROPOSITION 3:

With constant returns to scale, the state prices must be the same in all three economies, assuming that there is positive production in all three. As seen already in Figure 13, the budget set of each agent h is strictly bigger in the Arrow-Debreu economy than in the L-economy, which is strictly bigger than in the N-economy. Hence, either the equilibria are identical or the Arrow-Debreu equilibrium allocation Pareto dominates the L-economy equilibrium allocation, which in turn dominates the N-economy equilibrium allocation. ∎

D. The CDS-Economy

We introduce into the previous L-economy a CDS collateralized by X. Thus, we take $J = J^X \cup J^Y$, where J^X consists of a contract promising $(0, 1)$, and J^Y consists of contracts (j, j) as described in the Leverage economy above. As in the L-economy, we know that the only contract in J^Y that will be traded is $j^* = d_D^Y$.

As we saw in Section IIA, for interior production that leaves positive X and Y in the economy, there are two marginal buyers $h_1 > h_2$. Optimistic agents $h > h_1$ hold all the X and all the Y produced in the economy, selling the bond $j^* = d_D^Y$ using Y as collateral and selling CDS using X as collateral. Hence, they are effectively buying the Arrow U security (the payoff net of debt and CDS payment at state D is zero). Moderate agents $h_2 < h < h_1$ buy the riskless bonds sold by more optimistic agents. Finally, agents $h < h_2$ buy the CDS security from the most optimistic investors (so they are effectively buying the Arrow D).

The variables to solve for are the two marginal buyers, h_1 and h_2, the asset price, p, the price of the riskless bond, π_{j^*}, the price of the CDS, π_C, and production plans, (z_x, z_y). The system of equations that characterizes the equilibrium in the *CDS*-economy with positive production of Y is given by

$$(13) \qquad\qquad\qquad (z_x, z_y) \in Z_0$$

$$(14) \qquad\qquad \Pi = z_x + p\, z_y \geq \tilde{z}_x + p\tilde{z}_y, \quad \forall (\tilde{z}_x, \tilde{z}_y) \in Z_0.$$

$$(15) \qquad\qquad\qquad \pi_U \equiv \frac{p - \pi_{j^*}}{d_U^Y - d_D^Y} = 1 - \pi_C$$

$$(16) \qquad\qquad\qquad \frac{\gamma_U^{h_1}}{\pi_U} = \frac{d_D^Y}{\pi_{j^*}}$$

$$(17) \qquad\qquad\qquad \frac{\gamma_D^{h_2}}{\pi_C} = \frac{d_D^Y}{\pi_{j^*}}$$

$$(18) \qquad (1 - h_1)(x_{0^*} + \Pi) + (x_{0^*} + z_x)\,\pi_C + \pi_{j^*} z_y = x_{0^*} + z_x + p\, z_y$$

$$(19) \qquad\qquad h_2\,(x_{0^*} + \Pi) = \pi_C\,(x_{0^*} + z_x).$$

Equations (13) and (14) describe profit maximization. Equation (15) rules away arbitrage between buying the Arrow U through leveraging asset Y and through selling CDS, while using asset X as collateral, assuming that the price of X is 1. Equation (16) states that h_1 is indifferent between holding the Arrow U security (through buying asset Y and selling contract (bond) j^*) and holding the riskless bond j^*. Equation (17) states that h_2 is indifferent between holding the CDS security and the riskless bond. Equation (18) states that total money spent on buying the total available collateral in the economy should equal the optimistic buyers' income in equilibrium, which equals all their endowments and profits $(1 - h_1)(x_{0^*} + \Pi)$, plus all the revenues $(x_{0^*} + z_x)\pi_C$ from selling CDS promises backed by their holdings $(x_{0^*} + z_x)$ of X, plus all they can borrow $\pi_{j^*} z_y$ using their holdings z_y of Y as collateral. Finally, equation (19) states the analogous condition for the market of CDS, that is the total money spent on buying all the CDS in the economy, $\pi_C(x_{0^*} + z_x)$, should equal the income of the pessimistic buyers, $h_2\,(x_{0^*} + \Pi)$.

By plugging the expressions $p - \pi_{j^*} = \pi_U\big(d_U^Y - d_D^Y\big)$ and $\pi_U + \pi_C = 1$ from equation (15) into equation (18), and rearranging terms, we get

$$(20) \qquad (1 - h_1)(x_{0^*} + \Pi) = \pi_U\Big(x_{0^*} + z_x + \big(d_U^Y - d_D^Y\big) z_y\Big).$$

Combining equations (16) and (17) yields

$$(21) \qquad\qquad\qquad \frac{\gamma_U^{h_1}}{\gamma_D^{h_2}} = \frac{\pi_U}{\pi_C}.$$

It might seem that π_U, π_C are the appropriate state prices that can be used to value all the securities, just as $\gamma_U^{h_1}, \gamma_D^{h_1}$ did for the leverage economy. Unfortunately, this is not the case. There are no state prices in the CDS-economy that will value all securities. Of course we can always define state prices q_U, q_D that will correctly price X and Y. The equilibrium price p of Y and the price 1 of X give two equations that uniquely determine these state prices:

$$(22) \qquad\qquad\qquad p = q_U d_U^Y + q_D d_D^Y$$

$$(23) \qquad\qquad\qquad p_X = 1 = q_U + q_D.$$

Equations (22) and (23) define state prices that can be used to price X and Y, but not the other securities. If they priced all the other securities, then every agent would demand either the up or down Arrow security. But in CDS-equilibrium, some agents collectively must hold a portfolio paying off $z_y(d_D^Y, d_D^Y)$, since that part of the risky asset payoff cannot be tranched. Hence, π_U, π_C over-value Y and q_U, q_D over-value j^*. It follows that

$$(24) \qquad\qquad\qquad \frac{\gamma_U^{h_1}}{\gamma_D^{h_1}} > \frac{\gamma_U^{h_1}}{\gamma_D^{h_2}} = \frac{\pi_U}{\pi_C} > \frac{q_U}{q_D} > \frac{\gamma_U^{h_2}}{\gamma_D^{h_2}}.$$

As before, we can illuminate equilibrium using an Edgeworth box diagram in Figure 15. The complication with respect to the previous Figures 11 and 12 is that now there are four state prices to keep in mind. The state prices q_U, q_D determine orthogonal price lines, one of which must be tangent to the production possibility frontier at Q. The optimistic agents $h > h_1$ collectively own $(1 - h_1)Q$, indicated in the diagram. Consider the point x_1 where the orthogonal price line with slope $-q_D/q_U$ through $(1 - h_1)Q$ intersects the X line. That is the amount of X the optimists could own by selling all their Y. Similarly the point y_1 where the line intersects the Y line is the amount of Y the optimists could own by selling all their X.

Extend the line with slope $-\pi_C/\pi_U$ through the point x_1 in one direction until it hits the vertical axis at C_O and in the other direction until it hits the horizontal axis. This represents the final consumption optimists could get by tranching x_1, either selling it to buy the CDS, or using it as collateral to sell the CDS. By (24) this line is flatter than the price line.

The budget set of the optimists is far bigger than the tranching of x_1 allows. Scale up x_1 by the factor $\gamma_U^{h_1} + \gamma_D^{h_2} > 1$, giving the point x_1^*. That is how much riskless consumption those agents could afford by selling X (at a unit price) and buying the cheaper bond $\left(\text{at the price } \pi_{j^*} < d_D^Y\right)$. Now draw the indifference curve of agent h_1 with slope $-\gamma_D^{h_1}/\gamma_U^{h_1}$ from x_1^* until it hits the vertical axis. By equation (16), that is the budget trade-off between j^* and x_U. By (15) this line must pass through y_1 and meet the vertical axis at C_O. It shows that the optimists can obtain the same amount of the Arrow U by turning all their goods into y_1 and using that as collateral to sell the riskless bond j^*. Similarly, draw the indifference curve of agent h_2 with slope
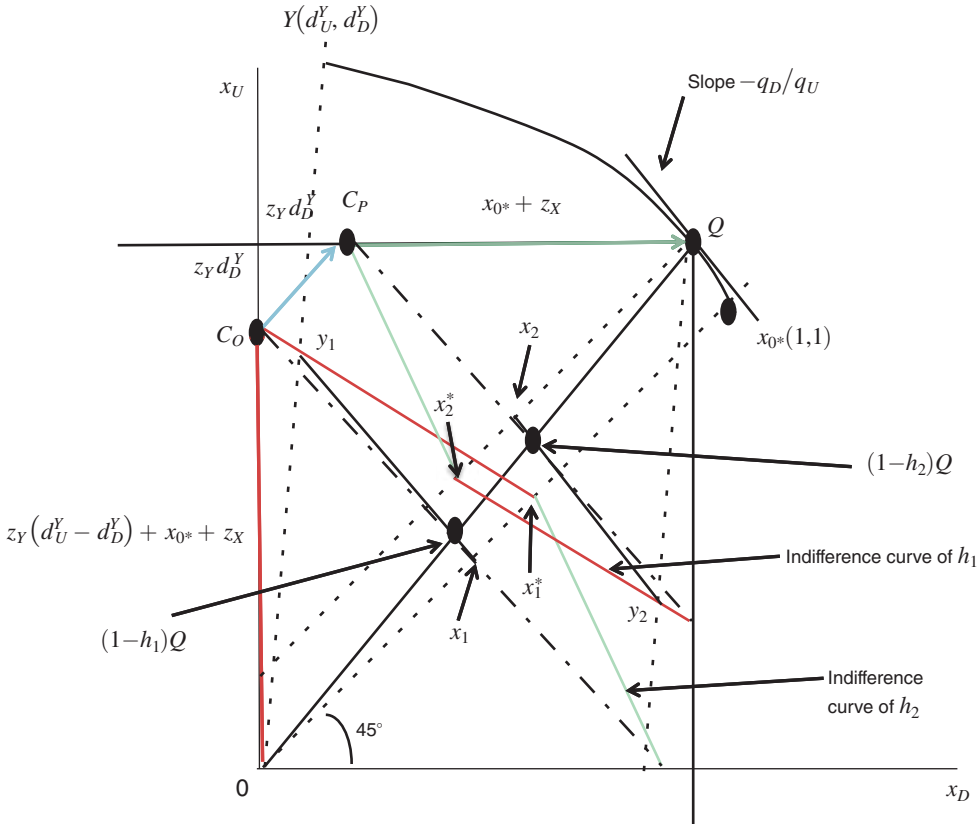
FIGURE 15. EQUILIBRIUM IN THE *CDS*-ECONOMY: EDGEWORTH BOX

$-\gamma_D^{h_2}/\gamma_U^{h_2}$ from x_1^* until it hits the horizontal axis. By equations (15)–(17), this is the same point that the tranching line hits the axis, and the new line represents the budget trade-off between j^* and x_D. These last two lines together form the collective budget constraint of the optimists. It is convex, but kinked at x_1^*. Notice that unlike before, the aggregate endowment is at the interior of the budget set (and not on the budget line). This is a consequence of lack of state prices that can price all securities. Because they have such flat indifference curves, optimists collectively will choose to consume at C_0, which gives $x_U = (x_{0*} + z_x) + z_y(d_U^Y - d_D^Y)$.

The pessimistic agents $h < h_2$ collectively own $h_2 Q$, which looked at from Q is indicated in the diagram by the point $Q - h_2 Q$. Consider the point x_2 where the orthogonal price line with slope $-q_D/q_U$ through $(1 - h_2)Q$ intersects the X line drawn from Q. Scale up that point by the factor $\gamma_U^{h_1} + \gamma_D^{h_2} > 1$, giving the point x_2^*. This represents how much riskless consumption those agents could afford by selling all their Y for X, and then selling X and buying the cheaper bond. The budget set for the pessimists, including the tranching line in the interior, can now be constructed as it was for the optimists, kinked at x_2^*. Pessimists collectively will consume at C_P, which gives $x_D = (x_{0*} + z_x)$. Finally, the moderate agents $h_1 < h < h_2$ collectively must consume $z_y d_D^Y$, which collectively gives them the 45-degree line between C_0 and C_P.

The picture can be derived from the equations, as we just saw. But one can also work in the opposite direction. For example, the convexity of the budget set makes clear which lines are steeper and guarantees the inequalities in (24). A simple geometric argument shows that because the budget lines meet at the 45° line at x_1^*, the triangle they form with the (π_U, π_C) line guarantees equation (21).

E. *CDS and Underinvestment: Proofs*

PROOF OF PROPOSITION 4:

The geometrical proof of Proposition 4 uses the Edgeworth box diagram for the *L*-economy in Figure 12 and is almost identical to that of Proposition 1. Consider an *L*-equilibrium in which not all of *X* is used in production. The optimists in the *CDS*-economy consume $z_y^{CDS}(d_U^Y - d_D^Y) + (x_0^* + z_x^{CDS})$ which is strictly more than in the *L*-economy as long as production is at least high in the *CDS*-economy. So suppose $z_y^{CDS} \geq z_y^L$ and $p^{CDS} \geq p^L$. This implies that $q_U^{CDS} \geq \gamma_U(h_1^L)$. By (24), $\gamma_U(h_1^{CDS}) \geq q_U^{CDS}$. Hence, $h_1^{CDS} \geq h_1^L$. By the same argument given in the geometrical proof of Proposition 1 in Section VC, consumption of the optimists in the *CDS*-economy cannot be higher than in the *L*-economy, which is a contradiction. Thus, either $z_y^{CDS} < z_y^L$ or $p^{CDS} < p^L$. But as we show in the Appendix B, profit maximization implies that if one inequality is strict, the other holds weakly in the same direction. ∎

PROOF OF PROPOSITION 5:

Let us begin with an interior CDS equilibrium. In Figure 15, define the point $(1 - k_1)Q$ as the intersection of the OQ line with a price line starting at C_O with slope $-q_D/q_U$. Notice that $(1 - k_1)Q$ is just to the southwest of $(1 - h_1)Q$. Similarly define the point $(1 - k_2)Q$ as the intersection of the OQ line with a price line starting at C_P with slope $-q_D/q_U$. Notice that $(1 - k_2)Q$ is just to the southwest of $(1 - h_2)Q$. Finally, define the point $(1 - k)Q$ as the intersection of the OQ line with a price line starting at the northwest corner of the Edgeworth Box with slope $-q_D/q_U$. At the prices (q_U, q_D), the pessimistic agents $h \in [0, k]$ can just afford to buy all the x_D in the economy, which costs $q_D z_Y d_D^Y$ more than C_P. At the prices (q_U, q_D), the optimistic agents $h \in [k, 1]$ can just afford to buy all the x_U in the economy, which costs $q_U z_Y d_D^Y$ more than C_O. It follows that $(k_1 - k)/(k - k_2) = q_U/q_D$.

Define h^* as the agent with $\gamma_U^{h^*} = q_U$. By the concavity of the function γ_U^h in h, it follows that

$$\frac{\gamma_U^{h_1} - \gamma_U^{h^*}}{\gamma_U^{h^*} - \gamma_U^{h_2}} \leq \frac{h_1 - h^*}{h^* - h_2}.$$

But by (24),

$$\frac{\gamma_U^{h_1} - \gamma_U^{h^*}}{\gamma_U^{h^*} - \gamma_U^{h_2}} = \frac{\gamma_U^{h_1} - \gamma_U^{h^*}}{\gamma_D^{h_2} - \gamma_D^{h^*}} > \frac{\gamma_U^{h_1}}{\gamma_D^{h_2}} > \frac{q_U}{q_D}.$$

It follows that $(h_1 - h^*)/(h^* - h_2) > q_U/q_D$. Hence, $(1 - h^*)Q$ lies just to the northeast of $(1 - k)Q$. It follows that at prices (q_U, q_D), the pessimistic agents $h \in [0, h^*]$ cannot afford to buy all the x_D in the CDS economy.

Suppose that investment in the CDS-economy is at least as high as in the Arrow-Debreu economy and that $q_U/q_D \geq q_U^A/q_D^A$. Then, $h^* \geq h_1^A$. Also, keeping the price of X at 1 in both economies, profits in the Arrow-Debreu economy are less, so wealth in the Arrow-Debreu economy is less than in the CDS-economy. Fewer agents $h \in [0, h_1^A]$ with less wealth each can buy less x_D in the Arrow-Debreu economy than the agents $h \in [0, h^*]$ in the CDS-economy. But since investment is less in the Arrow-Debreu, there is more remaining x_D in the Arrow-Debreu economy that needs to be consumed. This contradiction proves the proposition in the interior case.

Suppose now that in the CDS equilibrium all the X is used in production. Then, $h_2 = 0$. We can no longer be sure that $\pi_U = 1 - \pi_C$. The $(1 - \pi_C, \pi_C)$ line through x_1 may now lie strictly below the budget lines formed by the indifference curves of h_1 and $h_2 = 0$ meeting at the point x_1^*. If so, by raising π_C, the line will tilt until it hits the vertical axis at C_O. At this point we must have $\pi_U = 1 - \pi_C$. This same line will hit the horizontal axis to the left of the end of the budget set. The same geometric argument mentioned earlier shows that

$$\frac{\gamma_U^{h_1}}{\gamma_D^{h_2}} \geq \frac{\pi_U}{\pi_C} > \frac{q_U}{q_D} > \frac{\gamma_U^{h_2}}{\gamma_D^{h_2}}.$$

But now we can repeat the same proof, reaching a contradiction from the hypothesis that equilibrium output in the Arrow-Debreu economy is strictly less (and thus involves the retention of some X and all the Arrow-Debreu equations). ∎

It is worth pointing out that all our proofs remain intact if we allow positive endowments y_0^* of Y. The pure exchange economy is thus a special case of our model (obtained by taking $Z^h = \{0\}$).

APPENDIX A

TABLE A1—FINANCIAL INNOVATION AND TOTAL INVESTMENT IN Y, $-z_x$, FOR VARYING k

	L-economy	N-economy	Arrow-Debreu	CDS-economy
k				
1	0	0	0	0
1.1	0.43	0.34	0	0
1.2	0.60	0.46	0	0
1.3	0.73	0.54	0	0
1.4	0.83	0.60	0	0
1.5	0.92	0.65	0.21	0.14
1.6	1	0.69	0.48	0.33
1.7	1	0.72	0.68	0.48

TABLE A2—FINANCIAL INNOVATION AND WELFARE FOR VARYING h

	L-economy	N-economy	Arrow-Debreu	CDS-economy
h				
$h = 0$	1.000112835	1.000282087	2.400112835	2.471551724
$h_2^{CDS} = 0.348$	1.000112835	1.000282087	1.020297567	1.050666524
$h^{AD} = h^L = h^N = 0.3545$	0.999983529	0.824954173	1.000019677	1.0495
$h_1^{CDS} = 0.388$	1.072199212	0.86286584	1.07223797	1.050623002
$h = 1$	1.714268009	1.199938033	1.714329977	1.679771242

TABLE A3—CDS AND NONEXISTENCE

	N-output	Arrow-Debreu output	CDS-output	CDS volume
k				
1	0	0	0	0
1.1	0.37081	0	0	0
1.2	0.54768	0	0	0
1.3	0.69823	0	0	0
1.4	0.83664	0	0	0
1.44	0.8899	0	0	0
1.441	0.8913	0.0056	0.0035	0.9975
1.45	0.9031	0.0567	0.0359	0.9752
1.5	0.96825	0.3197	0.0209	0.8600
1.55	1.0323	0.5548	0.3734	0.7590
1.6	1.09552	0.77	0.5271	0.6705
1.65	1.1579	0.971	0.6722	0.5926
1.7	1.21958	1.1617	0.8098	0.5236

APPENDIX B

Profit Maximization.—In every equilibrium, each agent must be maximizing profit. Without loss of generality, we can suppose that every agent chooses the same production (z_x, z_y). Since by assumption the mass of agents is normalized to 1, total holdings in the economy are then $(x_{0^*} + z_x, z_y)$. Consider two asset prices p, q, and production plans $z^p = (z_x^p, z_y^p), z^q = (z_x^q, z_y^q)$ that maximize profits at the corresponding prices, so $z_x^p + p z_y^p \geq z_x^q + p z_y^q$ and $z_x^q + q z_y^q \geq z_x^p + q z_y^p$. Adding the inequalities and rearranging, $(p - q)(z_y^p - z_y^q) \geq 0$. So $p > q$ implies $z_y^p \geq z_y^q$, and $z_y^p > z_y^q$ implies $p \geq q$.

APPENDIX C

In the N-economy no financial contracts can be traded using the assets as collateral, and hence $J = \emptyset$. This situation corresponds to financial autarky. As before, given our assumptions, (z_x, z_y) is the aggregate production. In equilibrium there is a marginal buyer h_1. All agents $h > h_1$ use all their endowment and profits from production $x_{0^*} + \Pi$ and buy all the Y produced in the economy. Agents $h < h_1$ instead hold all the X in the economy. This equilibrium can be seen in an Edgeworth box as shown in Figure C1.
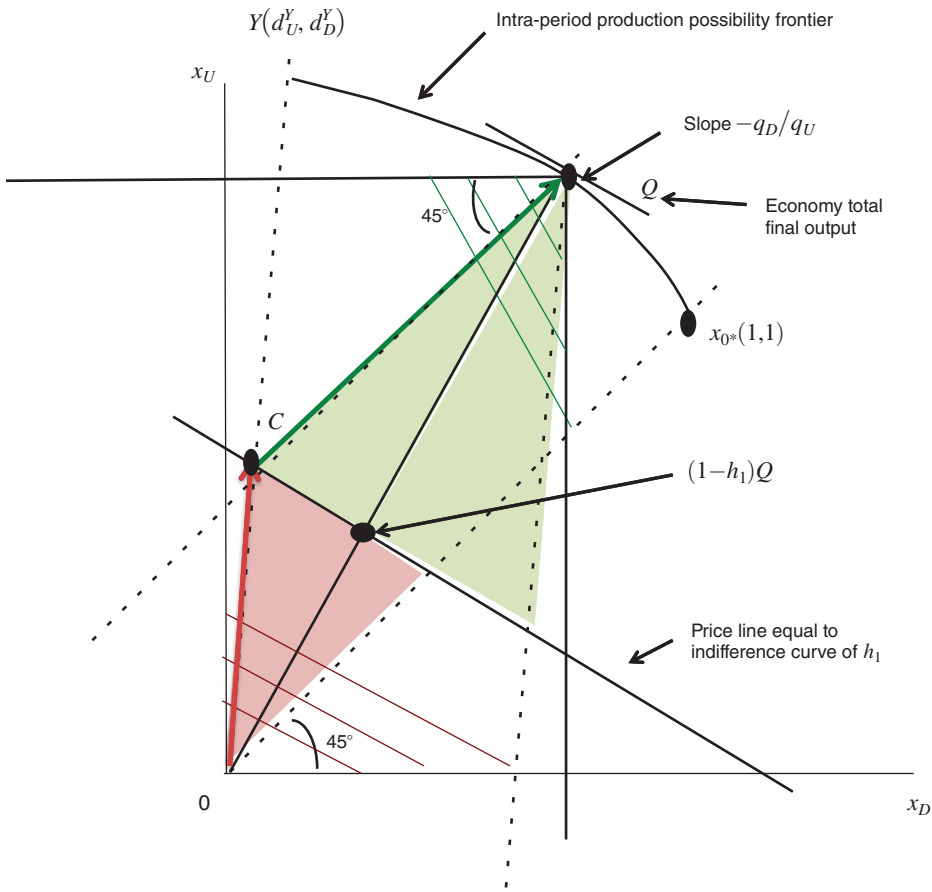
FIGURE C1. EQUILIBRIUM REGIME IN THE N-ECONOMY: EDGEWORTH BOX

The endogenous variables to solve for are the price of the risky asset p, the marginal buyer h_1 and production plans (z_x, z_y). The system of equations that characterizes the equilibrium in the N-economy is given by

$$(z_x, z_y) \in Z_0$$

$$\Pi = z_x + p z_y \geq \tilde{z}_x + p \tilde{z}_y, \forall (\tilde{z}_x, \tilde{z}_y) \in Z_0$$

$$(1 - h_1)(x_{0^*} + \Pi) = p z_y$$

$$\gamma_U^{h_1} d_U^Y + \gamma_D^{h_1} d_D^Y = p.$$

The first and second equations describe profit maximization. The third equation equates money $p z_y$ spent on the asset, with total income from optimistic buyers in equilibrium: all their endowment $(1 - h_1) x_{0^*}$ and profits from production $(1 - h_1)\Pi$. The final equation states that the marginal buyer prices the asset.

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