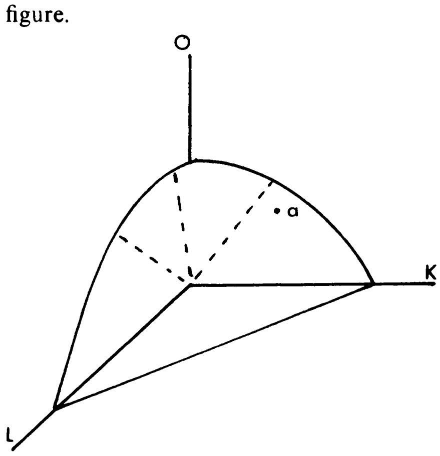
The 1975 Nobel Prize in Economics: Resource Allocation

The Nobel Prize in Economics for 1975 was awarded to Academician Leonid V. Kantorovich of the Academy of Sciences of the Soviet Union and Professor Tjalling C. Koopmans of the Cowles Foundation and the Department of Economics at Yale University for their contributions to the theory of optimum allocation of resources. This brief citation cannot reveal the magnitude of the scientific revolution in economic theory and methods initiated by their work. It is impossible, in a short essay, to give an adequate idea of the range of new mathematical techniques which were introduced, and the variety of problems which were studied, as this set of ideas unfolded. I shall, instead, attempt to give a brief discription of one of the significant conceptual innovations associated with their names—the replacement of what is known as the production function by the substantially more flexible considerations of activity analysis.

Let me illustrate the concept of a production function by an elementary example. Imagine a firm engaged in the production of a single output on the basis of certain factors of production. By an act of heroic abstraction and aggregation let us agree that there are only two factors, a generalized input of labor and a generalized capital. A specific technique of production would then be represented by three numbers: the first a quantity of output, the second the required input of labor for this quantity of output, and the third a corresponding figure for capital. Such a triple might be given by point *a* in the following figure.



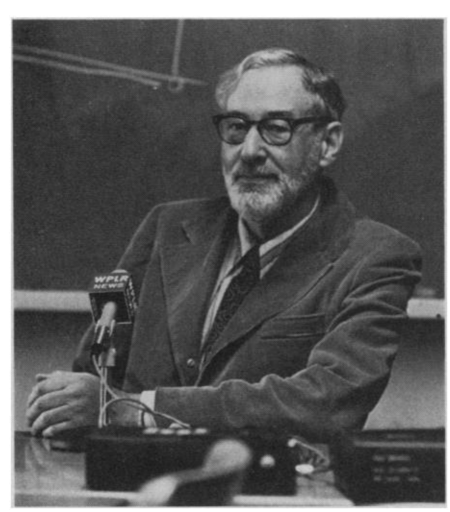
Typically, there will be many different combinations of capital and labor capable of producing the same quantity of output. The totality of these combinations can be described by a function whose independent variables are arbitrary quantities of labor and capital and whose dependent variable is the maximum quantity of output which can be obtained from these factors. The function in the foregoing figure represents such an example [output is represented by O, labor by L, capital by K, and the production function has the form $O = f(L, K)$].

The production function can be used quite readily to analyze the response of a profit-maximizing firm to a change in the price of one of its factors of production. Let the price of output be π_O, the price of labor (the wage rate) π_L, and the price of capital (there are considerable subtleties here) π_K. If the firm is sufficiently small that its choice of technique has no impact on these three prices, then the profit associated with a particular choice of labor and capital will be

$$\pi f(L,K) - \pi_L L - \pi_K K$$

The profit-maximizing firm will simply select that ratio of labor and capital which maximizes this function. If the price of one of the factors then changes, the maximizing ratio will shift in a way which can easily be predicted from the general shape of the production function without any need to exhibit its specific form.

Leonid V. Kantorovich



Tjalling C. Koopmans

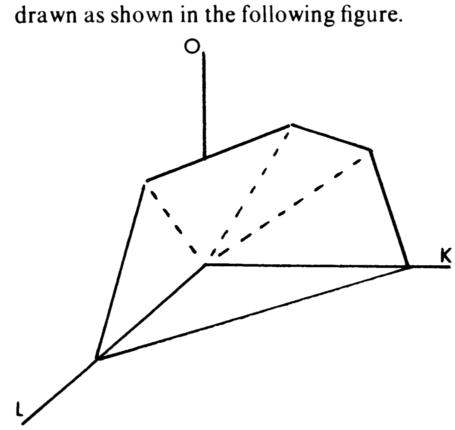
The activity analysis model permits the production function and its generalizations to be deduced from a prior specification of a finite list of basic activities. Each activity is described by a vector whose coordinates represent the inputs and outputs associated with the operation of a particular technique at a unit level. The fundamental assumptions of the activity analysis model are that the separate activities can be operated simultaneously and at arbitrary nonnegative levels.

For example, a firm producing a single output from labor and capital might have its technical capabilities described by an activity analysis matrix of the form

$$A = \begin{bmatrix} -1 & 0 & 0 & 1 & 1 & 1 \\ 0 & -1 & 0 & -4 & -3 & -2 \\ 0 & 0 & -1 & -2 & -4 & -7 \end{bmatrix} \begin{matrix} O \\ L \\ K \end{matrix}$$

Each column in this matrix represents a technology for producing output from labor and capital. Positive entries in the column represent outputs produced and negative entries inputs used up when the activity is operated at a unit level. The first three columns simply describe the possibility that each of the commodities may be disposed of at no cost.

It is assumed that each activity is capable of being operated at an arbitrary nonnegative level. If the six possible activity levels are given by $(x_1, x_2, \ldots x_6)$, then the set of possible production plans available to this firm will be the set of vectors in three-dimensional space represented by Ax, as x takes on all possible nonnegative values. The set of such plans may easily be drawn as shown in the following figure.



In this particular example the set of production plans afforded by the activity analysis specification is bounded from above by a surface which is virtually identical with the production function previously described. The major difference here is that instead of being smooth the upper surface

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An imaging system developed by G. L. Brownell and his associates at Massachusetts General Hospital can be used both for image reconstruction and conventional radionuclide imaging. Two-dimensional arrays each containing 127 detectors (scintillating crystals and photomultiplier tubes) sit on opposite sides of the patient. When the two arrays are used as a coincidence detector, the picture corresponds to a normal isotope scan. When the arrays rotate through 180 degrees, however, and readings are taken at intervals of 5 to 7 degrees, the information required for several cross sections is accumulated. So far, the Massachusetts General group has made cross sections of artificial test objects (phantoms) and of the brain in human subjects.

Researchers working under Victor Perez-Mendez at Lawrence Berkeley Laboratory, and under Leon Kaufman and Chang Lim at the University of California, San Francisco, have constructed and tested a positron annihilation image reconstruction system that involves no scanning motion at all. The reconstructed image, however, is not a true transaxial cross section, but is more akin to an earlier form of tomography known as focal plane imaging. According to Kaufman, the entire system could cost much less than the least expensive x-ray scanner.

The multiwire proportional chambers used in the coincidence detector are a major factor in the cost estimate. The detector is similar to the xenon detector that is expected to be used in some fast x-ray scanners, but the array is two-dimensional, covering an area 48 by 48 centimeters. Coincidence events are detected by elements in the two arrays opposite one another on either side of the patient.

Still other improvements to computerized scanners are being explored. Observers consider imaging techniques that do not impart the 2- to 4-rad dose of x-rays but retain the ability of x-ray scanners to make images with a high fidelity to be especially important because 2 to 4 rads is still too high for routine screening of nominally healthy persons. One way to reduce dosage is to find improved algorithms so that accurate images can be reconstructed from fewer data points. Research on combining image reconstruction from projections with ultrasound (which is considered safe because it produces no ionizing radiation) and with nuclear magnetic resonance are all under study for this reason.

Budinger, Kenneth Crowe, and their associates at the Lawrence Berkeley Laboratory have been studying the utility of radiography using helium ions and other so-called heavy charged particles as another approach to low dosage imaging. The interactions suffered by such particles do not result in their being absorbed as x-rays are, but are more like collisions between hard spheres in which a small amount of energy is lost in each collision. Thus, in heavy charged particle imaging, it is the energy of the transmitted particles rather than their number that is measured.

In other respects, imaging with heavy charged particles proceeds in the same way as with x-rays. Budinger, Crowe, and their colleagues have reported on cross-sectional reconstructions of the brain. Instead of a linear scanning motion, the investigators used a parallel beam of helium ions that is equivalent to the many parallel beams produced by the x-ray scanners. The method is not yet ready for clinical use, however, because the patient must be rotated in front of the beam which is obtained from the Berkeley 184-inch cyclotron.

—ARTHUR L. ROBINSON

Economics

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is composed of a finite number of linear segments. In this respect, however, the example is quite misleading. In most applications of activity analysis to the firm the matrix A will have many rows and columns—perhaps running into the thousands—with many possibilities of substitution, with an explicit account of intermediary goods, and with a great number of different outputs. As distinct from the description of production by means of one function, there will rarely be a single commodity whose output is being studied in isolation.

This point may be seen by examining a highly aggregated model of an economy along the lines of input-output analysis, a precursor of the activity analysis model introduced by Leontief in the 1930's. Let us imagine that the economy produces only two outputs, each of which can be consumed directly or used as an input into production, and let labor be the only scarce factor. An input-output table for this economy might be given by

$$\begin{bmatrix} 6 & -7 \\ -3 & 10 \\ -1 & -1 \end{bmatrix} \begin{matrix} O_1 \\ O_2 \\ L \end{matrix}$$

where I have specifically deleted the columns referring to costless disposal. In this example, and in its extensions to a more disaggregated model of the economy, there is no single output on which our attention is focused; the totality of all outputs is studied simultaneously.

An activity analysis model of an economy-wide production set would be considerably more general. Each output would be capable of being produced by more than one activity; there would be many scarce factors, such as machinery of various types and inputs of raw materials. We would be explicit in the representation of import and export possibilities open to the economy and might even consider having the model extend over time so as to explore its dynamic properties. The activity analysis model makes available a study of production which is of astonishing flexibility and generality.

Tjalling C. Koopmans, one of the two recipients of the Nobel award, was led to his introduction of the activity analysis model in the early 1940's by a study of the efficient use of transportation facilities. Koopmans, who was employed by the British Merchant Shipping Mission in Washington, D.C., during World War II, became concerned with the problem of selecting shipping routes to deliver a preassigned list of commodities to specified destinations in such a way as to minimize the total cost of shipping. Koopmans realized that the problem could be cast in the form of an activity analysis model in which each basic activity represented the selection of a particular shipping route. To take a simple example, consider a homogeneous commodity available at each of two locations, I and II, and which is required in definite amounts at each of three destinations, A, B, and C. The possibilities of shipping may then be described by the following activity analysis model

$$\begin{bmatrix} -1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & -1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ -5 & -3 & -7 & -10 & -20 & -8 \end{bmatrix} \begin{matrix} \text{I} \\ \text{II} \\ \text{A} \\ \text{B} \\ \text{C} \\ \text{Cost} \end{matrix}$$

in which the last row represents the cost of shipping a single unit from a given origin to a given destination.

In his memorandum on the transportation problem, Koopmans suggested a method of solution based on an economic idea which was to become of central importance in subsequent developments. He realized that a vector of prices—one at each location—would be associated with an optimal shipping plan. The prices would meet the condition that each route in use would make a profit of zero, in the sense that the price of the commodity at its destination would equal the price at its origin plus the unit shipping cost along that route. Moreover, the routes not being used would all make a profit less than or equal to zero. This permitted him to make the fertile suggestion that once the correct prices were known, the optimal selection of routes could be done in a decentralized fashion by managers who had in mind their own private considerations of profit maximization.

In 1944 Koopmans became a member of the Cowles Commission for Research in Economics, then located at the University of Chicago. Within a short period of time, Koopmans and his associates made the Cowles Commission into one of the major centers for the study of the optimal allocation of resources and its applications to a wide variety of economic problems.

In the late 1940's Koopmans extended his observation about the relationship between prices and optimality to the general activity analysis model. He demonstrated that any efficient production plan for such a model (a plan for which no alternative existed using less inputs and providing a greater output) would be associated with a vector of prices. The prices would yield a profit of zero for the activities composing the plan and a profit less than or equal to zero for all remaining activities. The insights about the decentralization of economic activity, in response to considerations of profit, were now available, not only for the transportation problem but for the functioning of an economic system in its entirety.

Many other remarkable innovations in economics were taking place at the same time. The modern study of the general equilibrium model, in which the theory of production is united with a description of consumer preferences, was inaugurated by Arrow, Debreu, and others; game theory, initiated by von Neumann and Morgenstern, made its appearance; the statistical estimation of complex economic relationships began to be studied; Arrow's book *Social Choice and Individual Values* was in the making; and Dantzig was developing the simplex method for linear programming problems which, in conjunction with the modern electronic computer, was to transform the activity analysis model into a tool for the solution of concrete economic problems.

But unknown to Western economists a corresponding development was taking place in the Soviet Union. In 1939 Leonid V. Kantorovich, the corecipient of the Nobel prize, presented a paper to a Leningrad seminar, whose contents were not available to colleagues in the West until some 20 years had passed. The paper, subsequently published under the title "Mathematical methods of organizing and planning production," provided its own formulation of the activity analysis model—somewhat different from that presented in the West—and outlined a computational technique for solving related linear programming problems. I find it difficult to appreciate the intellectual milieu in which the paper was written other than to see it as a response to specific practical questions addressed to its author. Kantorovich had been a pure mathematician whose previous publications gave no indication of an exposure to economic reasoning. It is all the more astonishing that, in the 1939 paper, the problem of the allocation of resources is treated not only from a mathematician's point of view but with a realization of the role played by prices in reaching an optimal decision. This paper and subsequent work by Kantorovich and his colleagues provided the base for the vigorous school of mathematical economics in the Soviet Union, which finally made its emergence in the late 1950's.

I have referred to linear programming without being explicit about the meaning of this term. Linear programming problems arise when the technical knowledge summarized by an activity analysis model is confronted with a given stock of factors of production and a specific objective to be maximized. A linear programming problem involves finding a nonnegative vector which satisfies a series of linear inequalities and which maximizes a given linear function. The first general formulation of these problems in the West was provided in 1947 by George B. Dantzig, currently Professor of Operations Research at Stanford University. At that time Dantzig and others were employed by the U.S. Air Force in the development of scientific programming techniques. Dantzig's concerns were with the explicit numerical solution of linear programming problems, and in the summer of that year he introduced the computational technique known as the simplex method. It is a remarkably effective algorithm, converging to the optimal solution in a relatively small number of iterations, even for problems of substantial size. Since that time Dantzig has remained the central figure in the development of a wide variety of mathematical programming methods and their application to problems of practical importance.

In my judgment, a computational technique which extends the range of applicability of economic theory—as the simplex method has done—is in itself a theoretical innovation of the highest order. This is particularly true in view of the close analogy, often cited by economists, between market mechanisms and formal mathematical techniques of optimization.

The techniques of activity analysis exemplify the pure theory of decision-making, and, as such, they are remarkably indifferent to economic institutions and organizational forms. For some economists, this is a deficiency of the theory of optimum allocation of resources: the forms in which economic activity is organized do matter. But on the other hand, one of the great achievements of this methodological revolution has been the continued dialogue—free of ideological overtones—between economists of the East and the West. This has been an event of considerable significance, and not only for economic reasons.

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Physiology or Medicine

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mammalian cells. A great deal is known about their DNA structure, function, and replication. Dulbecco's original goal of using a small DNA animal virus to study genetic processes is well on the way to realization. The role of these viruses in cell growth control is much less clear. Control of cell growth is itself a complex and subtle process and will require cooperative efforts from many different directions in order to be understood.

Howard Temin and David Baltimore were led to their simultaneous but independent discoveries of RNA-dependent DNA synthesis by RNA tumor viruses (reverse transcription) from quite different directions. Temin studied Rous sarcoma virus (RSV) when he was a graduate student at Caltech, working with Harry Rubin in Dulbecco's laboratory. In 1958 Temin and Rubin developed the first reproducible assay in vitro for a tumor virus, the focus-forming unit assay for RSV in chick embryo fibroblasts.

Temin continued to work with RSV after moving to Wisconsin in 1960, and was intrigued with the observations that inhibition of DNA synthesis, and inhibition of DNA-dependent RNA synthe-