

COMMENT ON: "ON THE STABILITY OF COMPETITIVE  
EQUILIBRIUM AND THE PATTERNS OF INITIAL  
HOLDINGS: AN EXAMPLE"\*

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When I first discovered the example of an economy for which the Walrasian price adjustment mechanism is unstable (published in this journal over twenty years ago) I was immediately confronted with questions about its realism and robustness. The example involved three commodities and three consumers, each of whose utility functions were of the form  $\min[\alpha_1 x_1, \alpha_2 x_2, \alpha_3 x_3]$ , but with extremely different parameters  $\alpha$  and vectors of initial holdings. The major virtue of the example was that the solution curves of the differential equations representing the price adjustment mechanism could be determined explicitly. The curves were closed orbits around the unique equilibrium price vector: if the initial price vector were not in equilibrium the solution would continue to oscillate forever.

Although this was the first counterexample to the conjecture that the Walrasian "tâtonnement" was always stable, it was far from ideal. Not only were the specific parameter values thoroughly unrealistic, but it was quite conceivable that an arbitrarily small perturbation would be sufficient to restore global stability. My own solution to this latter problem was given in the second part of the paper, in which a family of further examples with C.E.S. utility functions was presented. Even though the solution paths could not be determined explicitly, a qualitative analysis was sufficient to verify instability; in fact the paths explode away from the unique equilibrium rather than exhibiting the neutral behavior of the earlier example.

In a way, this is ancient history, given our current knowledge about the generality of market excess demand functions. The body of analysis, initiated by Sonnenschein and carried forward by Mantel, Debreu, and others, has shown us that market excess demand functions are essentially arbitrary, aside from the minimal considerations of continuity, homogeneity and the Walras Law. In mathematical terms the competitive model of exchange leads to a general vector field on the unit sphere, and examples can easily be given so that the solution of the Walrasian price adjustment mechanism is an arbitrary differentiable curve.

These considerations seem to me to provide the basic intellectual justification for the use of advanced mathematical techniques — such as fixed point theorems — in demonstrating the existence of equilibrium prices. They also indicate clearly that the problem of calculating equilibrium prices in a general equilibrium model is more complex than the corresponding problem on the production side alone.

But it is quite conceivable that the examples which defy elementary methods

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are basically so unrealistic that they would rarely be met in practice. For example, in calculating equilibrium prices, it may be a prudent strategy to try a simple technique like the price adjustment mechanism even though this is not guaranteed to converge with certainty. It is on this point that I find the analysis presented by Professor Hirota [this issue] to be quite significant in itself and potentially very stimulating for future research.

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#### REFERENCES

- HIROTA, M., "On the Stability of Competitive Equilibrium and the Patherns of Initial Holdings: An Example," *International Economic Review*, 22 (June, 1981), 461-467.